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A problem with the course presentation of the single-price alternative to 3rd-degree price discrimination

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ABSTRACT

The typical procedure for the determination of output, price, and profit associated with the single-price alternative to 3rd-degree price discrimination found in intermediate texts, managerial texts, and other texts concerned with pricing works well under certain specifications with respect to revenue and cost, but not all. It is over-simplified and, as such, unreliable for the determination of output, price, and profit, dependent on arbitrary, but specific, choices of values for the parameters of associated revenue and cost functions; and is, therefore, at least non-general. Suggestions for course presentation of the single-price alternative, given the reconsideration of the procedure for the determination of output, price, and profit and the computation of a discriminating critical value developed in this paper, are easily inferred. Illustrations are provided throughout.

1. INTRODUCTION

The benefit to a firm that engages in 3rd-degree price discrimination is often claimed to be made clear in course presentations by a comparison of results with respect to the output, price, and profit under price discrimination to such results associated with the single-price alternative to price discrimination. However, for the results associated with the single-price alternative to be appropriate for such a comparison in the establishment of the claimed benefit of 3rd-degree price discrimination, the results for the alternative must, of course, be reliable with respect to the profit-maximising levels of output, price, and profit under that alternative. In this regard, it is shown in this paper that the particular levels of output, price, and profit for the single-price alternative derived from the procedure typical of intermediate texts, managerial texts, and other texts concerned with pricing are unreliable, dependent on arbitrary, but specific choices of values for parameters of revenue and cost functions, and, therefore, at least non-general. See, for example, Baye (2006), Png and Lehman (2007), Keat and Young (2008), Hirschey (2009), Fisher *et al* (2010), and Thomas and Maurice (2010). Suggestions for course presentation of the single-price alternative, given the reconsideration of the procedure for the determination of output, price, and profit and the computation of a discriminating critical value developed in this paper, are easily inferred. Illustrations are provided throughout.

There is a considerable literature with respect to the welfare consequences of 3rd-degree price discrimination, in general, both pro and con. This is so, in particular, with respect to the welfare consequences for consumers in relatively small markets not otherwise served under the single-price alternative to 3rd-price discrimination that provides insight for this reconsideration.[2] In a paper by Layson (1994), following the work of Battalio and Ekelund (1972) in which a geometrical analysis is provided, algebraic conditions are offered for the determination of whether or not consumers in small markets otherwise not served under the single-price alternative would, indeed, be served under 3rd-degree price discrimination. Nevertheless, although such a systematic consideration of the specific conditions under which price discrimination would include consumers not previously served exists in the literature, and is well done, and despite previous qualifications, it has long been thought in the specific case of independent demands and constant marginal costs that firm output is invariant to the application of 3rd-degree price discrimination.[3]

Recent literature[4] that exists in particular with respect to course presentations of the consequences of the application of 3rd-degree price discrimination clearly indicates that the long-standing belief of invariance continues. The reconsidered method of solution for the levels of output, price, and profit presented in this paper for the single-price alternative, to which the results under price discrimination may be compared in a parameterised model, are indicated, below. Such a reconsidered method of solution is more thorough than the typical method of solution common to the various microeconomic texts.

2. Initial conditions and standard practice

In standard practice, to establish a basis for comparison, consider conditions in which the demands in markets, R and S, are given, for example, by specific, parameterised equations, such as:

$$p_r = 12 - 0.5 * q_r \tag{1}$$

and

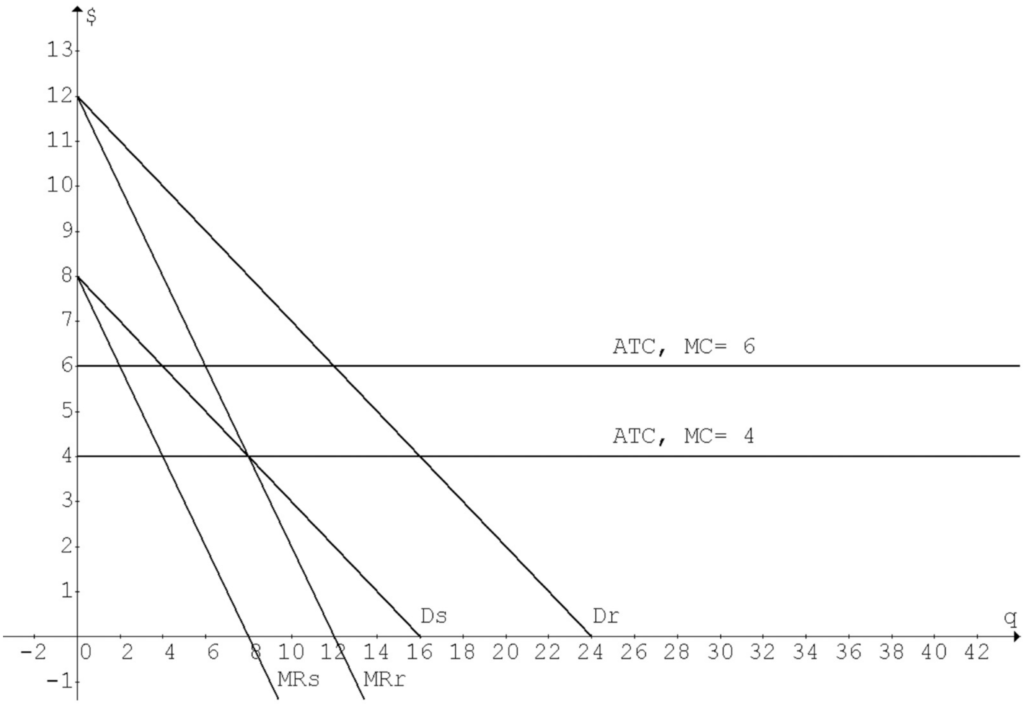
$$p_s = 8 - 0.5 * q_s \tag{2}$$

where pr and ps are prices per unit and qr and qs are individual market quantities in markets, R and S, respectively.[5] For ease of exposition, let average and marginal cost be equal to a constant, K. As such, associated profit under 3rd-degree price discrimination, $\pi_{discrimination}$, is defined by the equation:

$$\pi_{discrimination} = pr * qr + ps * qs - K * (qr + qs) \quad (3)$$

See Figures 1, 2A and 2B for illustration. With respect to Figure 2A, a successful implementation of 3rd-degree price discrimination is shown to exist. Note the various solutions, described below.

Figure 1: Profit Maximisation in Individual Markets



For profit-maximisation under 3rd-degree price discrimination, the partial derivatives of the defined profit function with substitution:

$$\pi_{discrimination} = (12 - 0.5 * q_r) * q_r + (8 - 0.5 * q_s) * q_s - K * (q_r + q_s), \quad (4)$$

with respect to q_r and q_s, are set equal to zero and are as given below:

$$\partial \pi_{discrimination} / \partial q_r = 12 - q_r - K = 0 \quad (5)$$

and

$$\partial \pi_{discrimination} / \partial q_s = 8 - q_s - K = 0. \quad (6)$$

Solving:

$$q_r = 12 - K \quad (7)$$

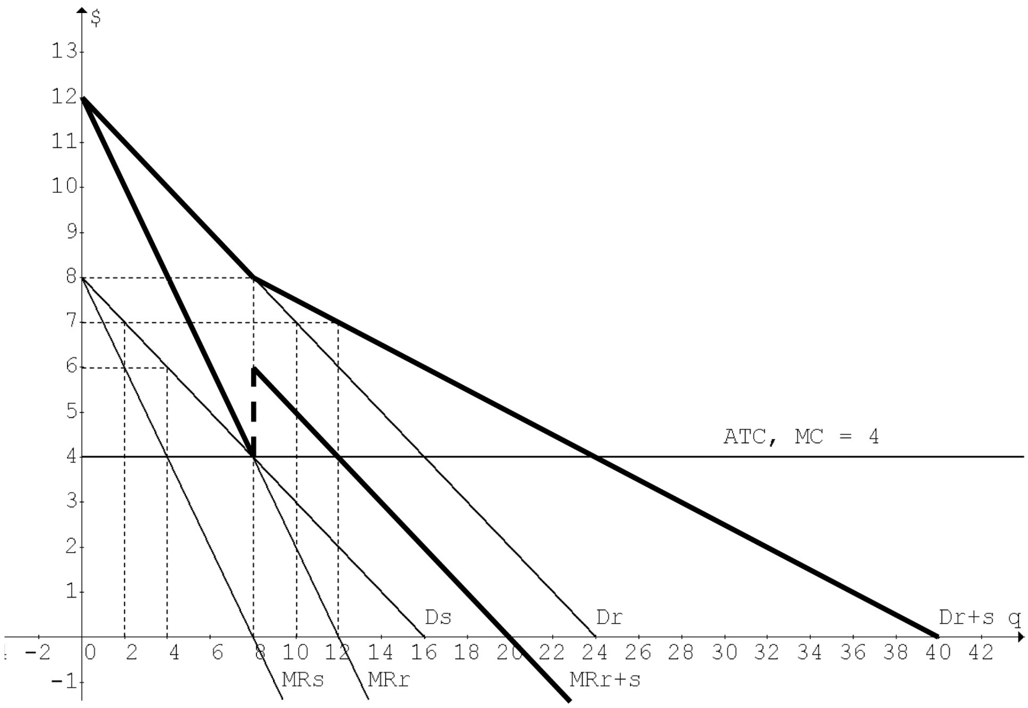
and

$q_s = 8 - K,$ (8)

provided that q_r and q_s are equal to or greater than 0. For example, where $K = 4$, then:

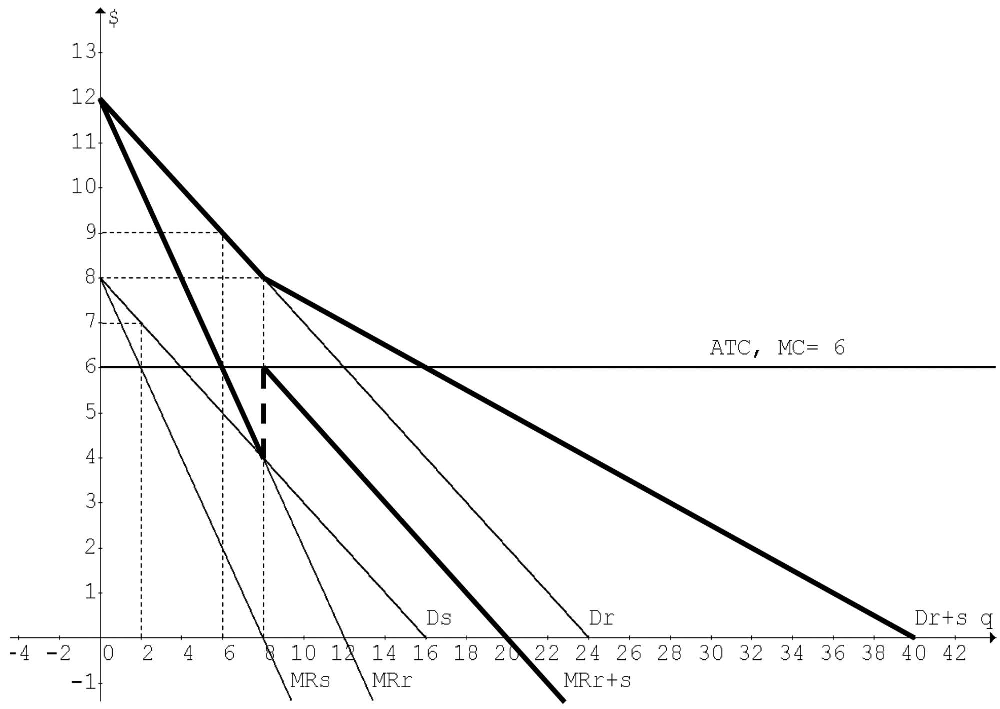
$q_r = 8$ and $q_s = 4$. Where K = 6, then $q_r = 6$ and $q_s = 2$.

Figure 2a



Graphically, individual market demand curves and associated individual marginal revenue curves are, respectively, horizontally summed as indicated in Figures 2A and 2B, where ATC and MC are each equal to 4 and to 6, respectively. At the intersection of the marginal cost curve and the summed marginal revenue curve, MRs+r, the total quantity of output is indicated. Individual market quantities are, then, determined by equating respective marginal revenue curves with marginal cost at the total quantity of output, previously specified. Prices are, then, determined by reference to individual demand curves at individual market quantities, respectively. Given the values for q_r and q_s, where $K = 4$, for example, $p_r = 8$ and $p_s = 6$. Where $K = 6$, then $p_r = 9$ and $p_s = 7$. Profit is computed as indicated, above, by equation (3). As such, where $K = 4$, for example, $\pi_{discrimination} = 40$. Where $K = 6$, then $\pi_{discrimination} = 20$. Again, see Figures 2A and 2B for illustration.

Figure 2b



3. A PERFUNCTORY DESCRIPTION OF THE TYPICAL SINGLE-PRICE SOLUTION

For the typical single-price solution found in intermediate texts, managerial texts, and other texts concerned with pricing as an alternative to 3rd-degree price discrimination, where associated profit is represented by the symbol, $\pi_{typical\ single}$, profit is given by the expression:

$$\pi_{typical\ single} = (12 - 0.5 * q_r) * q_r + (8 - 0.5 * q_s) * q_s - K * (q_r + q_s). \quad (9)$$

The partial derivatives of the Lagrange-expressed profit function:

$$\pi_{typical\ single} = (12 - 0.5 * q_r) * q_r + (8 - 0.5 * q_s) * q_s - K * (q_r + q_s) - \lambda * ((12 - 0.5 * q_r) - (8 - 0.5 * q_s)) \quad (10)$$

with respect to q_r, q_s, and λ, the Lagrange multiplier, set equal to 0 are as given below:

$$\partial\, \pi_{typical\ single}/\partial\, q_r = 12 - q_r - K + \lambda * 0.5 = 0, \quad (11)$$

$$\pi_{typical\ single}/\partial\, q_s = 8 - q_s - K - \lambda * 0.5 = 0, \tag{12}$$

and

$$\pi_{typical\ single}/\partial\, \lambda = 12 - 0.5 * q_r - 8 + 0.5 * q_s = 0. \tag{13}$$

The Lagrange-expressed profit function, eq. (10), sets $\pi_{typical\ single}$ equal to the sum of revenues less total cost, as defined in eq. (9), and embodies the constraint that the prices in each market must be equal, that is, that (12 - 0.5 * q_r) - (8 - 0.5 * q_s) = 0. Solving:

$$q_r = 14 - K \tag{14}$$

and

$$q_s = 6 - K \tag{15}$$

provided that q_r and q_s are equal to or greater than 0. As such, where $K = 4$, for example:

$q_r = 10$ and $q_s = 2$. Where $K = 6$, then $q_r = 8$ and $q_s = 0$.

As indicated, above, individual market demand curves and associated individual marginal revenue curves are, respectively, horizontally summed, as indicated in Figures 2A and 2B. At the intersection of the marginal cost curve and the summed marginal revenue curve, the total quantity of output can be specified. The particular value of price for the single-price alternative to 3rd-degree price discrimination derived from this procedure common to the various microeconomic texts is then determined by reference to the summed demand curve, Dr+s, at the total quantity of output. Given the value for the sum of q_r and q_s, where $K = 4$, price in this single-price case is equal to 7. Where $K = 6$, price in this single-price case is equal to 8. See Figures 2A and 2B. Profit is computed as indicated, above, by equation (9). As such, where $K = 4$, for example, note that $\pi_{typical\ single} = 36$. Where $K = 6$, then note that: $\pi_{typical\ single} = 16$. Although this typical procedure often ends with such a result as if it is determinant and final, it is not necessarily thorough and complete. Consider a more thorough procedure and a substitute for the determination of the typical profit-maximising single-price solution, below.

3. A more thorough single-price method of solution as an alternative to 3rd-degree price discrimination

For the more thorough single-price method of solution as an alternative to 3rd-degree price discrimination, maximum profit is determined on the basis of a comparison of the level of profit associated with the typical method of solution, above, and the level of profit derived when one market only, that is, market R in this example, is served. Associated profit is represented by the symbol, $\pi_{typical\ single}$. The derivative, and this is the important insight in this paper, set equal to 0, as a first consideration, of the one-market profit function:

$$\pi_{alternative\ single} = (12 - 0.5 * q_r) * q_r - K * q_r, \tag{16}$$

with respect to q_r, only, is as given below:

$$\partial \pi_{alternative\ single} / \partial q_r = 12 - q_r - K = 0. \tag{17}$$

Solving:

$$q_r = 12 - K, \tag{18}$$

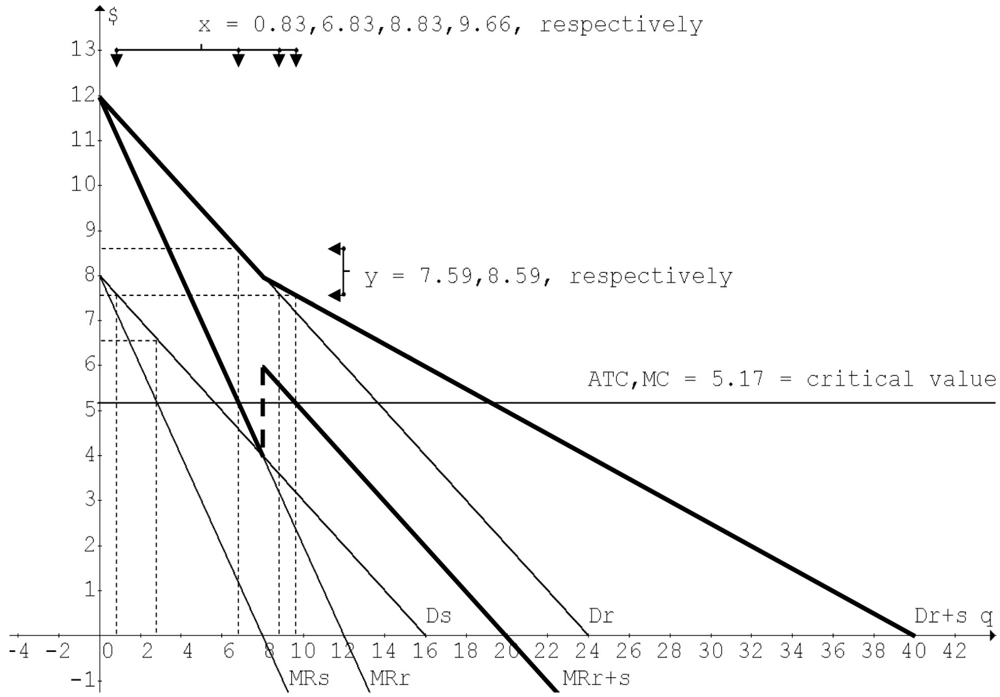
provided that q_r is equal to or greater than 0. As such, where $K = 4$, for example, $q_r = 8$. Where $K = 6$, then $q_r = 6$.

At the intersection of the marginal cost curve and the individual marginal revenue curve, MRr, the total quantity of output is indicated. The particular value of price for the single-price alternative to 3rd-degree price discrimination derived from this first consideration of this more thorough method of solution is then determined by reference to the associated demand curve, Dr, at the total quantity of output. Given the value for q_r, where $K = 4$, price in this single-price case is equal to 8. Where $K = 6$, price in this single-price case is equal to 9. See Figures 2A and 2B. Profit is computed as indicated, above, by equation (9). As such, where $K = 4$, then:

$\pi_{alternative\ single} = 32$. Where $K = 6$, then $\pi_{alternative\ single} = 18$.

Note, therefore, that by comparison the typical single-price solution is shown to be unreliable as a measure of the level of maximum profit obtainable for a single-price alternative, in that respective values for the sum of q_r and q_s from the typical solution and for q_r , only, from the substitute solution, result in profits that are greater, i.e., $\pi_{typical\ single} = 36$ is greater than $\pi_{alternative\ single} = 32$, where $K = 4$, but less, i.e., $\pi_{typical\ single} = 16$ is less than $\pi_{typical\ single} = 18$, where $K = 6$, than that associated with the substitute single-price solution dependent, of course, on the specific choice of values for the parameters of the cost function, i.e., for values of K, given demands for markets, R and S. As such, where $K = 4$, the typical single-price solution results in $\pi_{typical\ single} = 36$, a value greater than that associated with the alternative single-price solution that results in $\pi_{typical\ single} = 32$, at most. Where $K = 6$, however, $\pi_{typical\ single} = 16$ for the typical single-price solution, a value less than that associated with the substitute single-price solution that results in $\pi_{alternative\ single} = 18$, where q_r, only, is produced. As such, the typical single-price solution and the substitute single-price solution are alternatively unreliable with respect to the level of maximum profit, dependent on the arbitrary, but specific choice of values for K for the cost function, given demands for markets, R and S, and, therefore, at least non-general. More generally, and as instruction for course presentation, where $\pi_{typical\ single}$ is greater than $\pi_{alternative\ single}$, take the typical single-price solution as the solution for the more thorough single-price solution. Where $\pi_{typical\ single}$ is less than $\pi_{alternative\ single}$, replace the typical single-price solution as the solution with the substitute single-price solution for the more thorough single-price solution.[9]

Figure 3



4. THE CRITICAL VALUE

Where the profit for the typical single-price solution is given by the equation:

$$\pi_{typical\ single} = K^2 - 20 * K + 100, \tag{19}$$

by substitution of equations (14) and (15) into equation (9), and the profit for the alternative single-price solution is given by the equation:

$$\pi_{alternative\ single} = 0.5 * K^2 - 12 * K + 72, \tag{20}$$

by substitution of equation (16) into equation (15), profits, respectively, are equal where:

$K^2 - 20 * K + 100 = 0.5 * K^2 - 12 * K + 72$ or, more conveniently, where:

$0.5 * K^2 - 8 * K + 28 = 0.$ (21)

Solving the quadratic for the critical value for K:

$K = 5.17.$[10] (22)

With respect to the results of the single-price alternative to 3rd-degree price discrimination, profit for the typical single-price solution is greater than profit for the substitute single-price solution for values of average and marginal cost set near, but less than, 5.17: and less than profit for the substitute single-price solution for values of average and marginal cost set near, but greater than, 5.17. Profits are equal, of course, for average and marginal cost set equal to 5.17. As such, the textbook method of solution typically presented for the single-price alternative to price discrimination is, indeed, unreliable with respect to the determination of maximum profit for values for marginal cost set near, but greater than 5.17 for the example referenced above. The acceptability of the typical method of solution is, in general, sensitive to the choices of values for parameters of the revenue and cost functions for problems presented. Sometimes it is correct, sometimes it is not.

By observation of Figures 2A and 2B it is obvious, where average and marginal cost are greater than 6, but even less than 8, that the typical method of solution for single-price results is unreliable with respect to the level of maximum profit. The output and price results, i.e. $4 < q < 8$ and $10 > p > 8$, imply the relevance of market R, only, even though output is specified at the intersection of the marginal cost curve and the summed marginal revenue curve, MRs, under the typical method of solution. It is not obvious, however, where average and marginal cost are less than 6, but greater than 5.17, that the typical method of solution for single-price results is unreliable with respect to the level of maximum profit, even though it is, indeed, unreliable.

In the absence of knowledge of such a critical value of K, it is unreliable to present the typical single-price solution as a correct solution with respect to the determination of the level of firm output, price, and maximum profit. In such an absence, typical and substitute single-price solutions as alternatives to solutions under 3rd-degree price discrimination must otherwise, themselves, be compared as a first step of a more thorough method of solution. If $\pi_{typical\ single}$ is greater than $\pi_{alternative\ single}$ as a first step, then the typical method of solution is acceptable as an appropriate method of solution. If $\pi_{typical\ single}$ is less than $\pi_{alternative\ single}$ as a first consideration, then the typical method of solution is unacceptable as an appropriate method of solution. Under such a condition, the more thorough method of solution should be used as the reliable single-price alternative for comparison to the results under price discrimination. Although different texts consider different comparisons to the single-price alternative dependent on the number of markets served, that presumption about the number is premature to the arbitrary, but specific choice of values of parameters.

Therefore, even though a reliable solution can be computed in the absence of knowledge of such a critical value of *K* by comparisons as indicated above, knowledge of the critical value of *K* provides the opportunity for a direct and efficient determination of a reliable solution. For MC less than the critical value, use the typical method of solution. For MC greater than the critical value, use the substitute method of solution.

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Endnotes

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2, For example, see Smith and Formby (1981), Varian (1985), Stigler (1987), and Hausman and Mackie-Mason (1988).

3. Although there are, indeed, several degrees of price discrimination, the analysis in this paper is limited, of course, to 3rd degree price discrimination. In particular, see Smith and Formby (1981).

4. For example, see Carroll and Coates (1999), Jeitschko (2001), Kwon (2006), Round and McIver (2006) and Hirschey (2009).

5. Although the standard practice of parameterised equations has been followed, a generalised version will be made available as an electronic appendix on the journal website.

6. Many other parameterised equations for demands in the various markets may, of course, be considered. For such other parameterised equations, critical values, discussed later in this article, can be computed by analogy.

7. The horizontal summation of the marginal revenue curves is not required given that marginal cost is assumed in this paper to be equal to a constant, *K*. The quantity produced for sale to any one group does not affect the marginal cost of the quantity produced to any other group. Nevertheless, it is typical of texts to do so.

8. Total output under both price discrimination and the single-price alternative to price discrimination are typically observed to be equal. Justifications of this result are made by reference to the condition that marginal cost is equal under each pricing condition. This, too, is unreliable with respect to the level of maximum profit, dependent on arbi-

trary, but specific choices of values for parameters of revenue and cost functions and, also therefore, at least non-general.

9. Note that the typical single-price solution, unreliable for the determination of the level of output and maximum profit for the single-price alternative to which the results under price discrimination may be compared, is logically inconsistent with respect to the conditions for profit maximisation. Where $K = 6$ and q_r is set at 8, associated marginal revenue, MRr, is less than marginal cost.

10. Extended, critical $K = 5.17157287525381$.

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