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Personality and Patterns of Savings: the Theory of Economic Growth beyond Optimal Behaviour

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ABSTRACT

In optimal growth models, in which the goal of the representative agent is to maximise intertemporal utility, the time trajectories of consumption and savings are endogenously determined. Deviations from such trajectories are not desirable and the rational economic agent will act with the purpose of avoiding them. Despite this logical argument, empirical evidence reveals a wide diversity of savings behaviour across the population: individuals with similar initial conditions and facing identical constraints often adopt savings patterns that are far from being coincidental. Such observation suggests that personality traits interfere in savings decisions, and eventually divert these decisions from those leading to purely optimal outcomes. Even when individuals know the optimal solution, their personality may compel them to act as savers or spenders, to a greater or lesser extent. This paper explores the impact and implications of different personalities in shaping savings and consumption trajectories, in the context of a standard growth model. Analytical results are derived for models of neoclassical growth and for models of endogenous growth. In a first stage, a standard infinite horizon scenario is considered. Subsequently, this is complemented by a setting of finite life cycles, where growth outcomes of a given generation are likely to be influenced by the consumption-savings decisions of the precedent generation.

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1. INTRODUCTION

The benchmark model in economic growth analysis, built upon the foundational work of Ramsey (1928), Cass (1965), and Koopmans (1965), consists of an optimal control problem, through which a representative agent maximises consumption utility. The problem is solved at some initial date, given an infinite horizon, and optimal time trajectories for consumption and savings are then derived. Although the growth model has evolved over time to include many relevant features (most noticeably the possibility of sustained endogenous growth: Romer 1986, 1990; Lucas 1988), its basic structure remains intact and continues to provide relevant insights on how consumption-savings choices by a rational agent shape the accumulation of wealth and the pace of growth (Akcigit 2017).

A notable feature of the standard growth model, as characterised above, is that the representative agent that populates the economy has no personality. Typically, the agent is constricted to act in a pre-specified optimal way, and almost no degree of freedom exists that allows for deviations from 'normal' behaviour. The only choice the agent often has consists in selecting the rate of time preference that best characterises her subjective degree of impatience. Beyond this, there is nothing else suggesting that different agents solving the growth model would, eventually, trigger different growth outcomes. In this paper, the personality of the agent is assumed to matter at a fundamental level: personality influences savings behaviour, leading to a possible deviation of the agent's choices relatively to the optimality benchmark.

Although they both exert influence on consumption and savings decisions over the life cycle, traits of personality and the rate of time preference play fundamentally different roles in the growth model. The intertemporal discount rate is an integrating component of the optimal planning problem that the agent solves, and it translates a desired degree of impatience selected to govern the choices of the representative agent over the assumed time horizon. Personality is introduced in the model with a different perspective: it constitutes an involuntary force that may compel the agent to deviate from optimal planning and to select a path for the savings rate that does not concur with the level of impatience the agent had initially and conscientiously established to represent time preference.

Under the above interpretation, personality might be associated with issues of self-control (Gul and Pesendorfer 2001; Krusell *et al* 2010; Pavoni and Yazici 2017; Kovacs *et al* 2021). In the proposed personality setting, self-control biases may follow in potentially opposite directions: they may signify an urge for an agent who intended to be patient to spend too much too fast; or, alternatively, a compulsion for someone who planned to spread consumption over time to save more than the optimal plan tells the agent to do (this is the behaviour adopted by miserly people). Personality might distort optimal plans, leading to outcomes that would never be experienced under optimal choices, regardless of the selected rate of time preference. One such outcome is the possibility of the emergence of dynamic inefficiency, as remarked upon with the development of the model in section 3.

Empirical literature pointing to a close relationship between agents' personality and savings behaviour is abundant. For instance, Cronqvist and Siegel (2015), resorting to data on twin brothers and twin sisters, find evidence that individuals with identical familiar and social background tend to adopt dissimilar savings attitudes, what can be explained by innate genetic predisposition and, hence, the personality each individual agent is revealed to possess. These authors stress the enormous observed differences of wealth accumulated at retirement age by individuals with identical initial conditions and similar income paths, and conclude that these cannot be explained solely by chance or by a different ability in allocating assets. On the contrary, such differences are mostly attributable to personality profiles: some of us are 'savers' (starting saving a large portion of income early in life) while others have the personality of 'spenders' (making consumption approach as closely as possible the trajectory of income).

In the same vein, Gerhard *et al* (2018) explore the link between psychological characteristics and household savings behaviour, through the analysis of the results of an inquiry answered by a representative sample of households. The fundamental conclusion is that there is a wide array of psychological variables conditioning savings and that these variables may have a distinct impact in different stages of the agents' life cycle. Among the long list of psychological drivers of savings, the authors strongly highlight the big five personality traits of psychological analysis. These traits, typically known by their acronym, OCEAN, involve five related but separate human characteristics, which allow to define the personality profile of a given individual. The traits are openness to experience, conscientiousness, extraversion, agreeableness, and neuroticism (see Digman 1990; Goldberg 1990; Costa and McCrae 1992; John and Srivastava 1999; McCrae and Costa 2008).

Invoking their own study, and previous work searching for the interplay between the big five and savings behaviour (Nyhus and Webley 2001; Brown and Taylor 2014; Cobb-Clark *et al* 2016; Mosca and McCrory 2016), Gerhard *et al* (2018) conclude that extraversion, agreeableness, and neuroticism are negatively correlated with a 'saver' attitude, while openness to experience and conscientiousness are positively correlated with the same 'saver' worldview. The results are intuitive: for instance, agreeableness and extraversion are traits that lead people to adopt pro-social attitudes and to be outward-oriented, features that are not compatible with a desire to accumulate or store a large slice of the earned income. In opposition, conscientious individuals will certainly adopt a careful attitude regarding their spending and therefore they will reveal a stronger propensity to save.

Many other studies have approached the interconnection between psychology and savings. Some of this literature includes: Wärneryd (1989), van Veldhoven and Groenland (1993), Thaler (1994), Shim *et al* (2012), Brounen *et al* (2016), Nyhus (2017), Asebedo *et al* (2018), Fuchs-Schundeln *et al* (2020), and Gomes (2021). Attaching savings to psychology is not, in fact, a novelty in Economics. Keynes (1936) enunciated a series of eight motives to save, most of which clearly go beyond the domain of pure economic reasoning, for example the improvement or independence motives or, most noticeably, the case of avarice (to the eight original motives, Browning and Lusardi, 1996, added one more – the downpayment motive).

However, regarding economic analysis, mostly at the macro and growth levels, theory continues to be based on a strict rationality interpretation of the reasons why agents eventually defer consumption and, thus, save. In modern growth theory, although agent heterogeneity has been introduced at a variety of levels (e.g., skills, endowments, or preferences: Perla and Tonetti 2014; Akcigit and Kerr 2018; Grossman and Helpman 2018; Perla *et al* 2021), there is still a strong reluctance to recognise that agents are heterogeneous regarding the way they approach savings decisions.

Recognising that the financial decisions of households (that is to say consumption-savings decisions) are, today, more complex, interdependent, and heterogeneous than ever (Gomes *et al* 2021), it becomes relevant to bring complexity, interdependence and heterogeneity into benchmark economic models. This is the goal pursued in this research, as it concerns the implications for the modelling of economic growth. In the sections that follow, the standard growth model is modified to explore the impact of the personality of agents on savings and growth. Transitional dynamics and steady state results are explored, in the context of neoclassical and endogenous growth models, taking into consideration both settings with a representative agent and settings involving heterogeneous interacting agents. Furthermore, infinite and finite horizons are considered. In the infinite horizon setup heterogeneous agents (i.e. agents with different personalities) may coexist, while in the finite horizon framework heterogeneity emerges in the form of sequential generations eventually endowed with divergent personalities (see Gomes 2022).

The main argument employed to modify standard growth models through the study is that agents are rational, have information, and are endowed with the capacity to solve the optimal growth model, but they choose not to do so. Instead, they adopt a 'saver' or 'spender' attitude, which they know will not lead to the optimal outcome. Given their psychological features, agents cannot avoid not acting optimally, even when they are fully aware of their behaviour. Results reveal deviations relative to the optimality benchmark and they are compared, on each occasion, with the optimal results that are standard in neoclassical and endogenous growth theories.

The remainder of the paper is organised as follows. Section 2 recovers the structure of the standard infinite horizon optimal growth model and makes a brief characterisation of it. Section 3 introduces the possibility of non-optimal savings behaviour, by proposing an exogenous savings rate, in the spirit of the Solow (1956) – Swan (1956) growth model. This exogenous savings rate, however, will be formulated with reference to the optimality benchmark, namely the optimal paths of capital and consumption. In this section, results are obtained for the neoclassical version of the growth model. An important point to discuss in this context is related with the notions of dynamic inefficiency and golden rule (Phelps 1966), and how deviating savings behaviour impacts the model regarding these issues.

In section 4, the steady state and transitional dynamics analysis of the effects of 'saver'/'spender' behaviour is extended to an endogenous growth model, more precisely to a straightforward AK model. In this case, savings behaviour will influence not only the steady state level of the main economic aggregates, but also their growth rates. Section 5 employs the AK growth model to highlight the implications of heterogeneity and interdependence. Groups of agents endowed with distinct personalities are assumed to populate the economy simultaneously. Such groups will contribute to the same production process and the growth outcome will emerge from the interplay between the groups' behaviours.

Section 6 presents a finite horizon model. In this scenario, it is possible to derive expressions explicitly for the evolution of capital, income, consumption, and savings over time. This makes it easier to understand how distinct attitudes towards savings shape specific trajectories of aggregate variables. Various assumptions might be taken regarding savings when assuming a finite horizon: while in Section 6 it is assumed that agents adopt a transversality condition of zero terminal savings regardless of their saving attitude across the life cycle, in Section 7 intergenerational altruism is considered, and in this case the savings attitude will effectively be an attitude of saving or spending having in perspective the welfare of the generations that follow. Section 8 concludes.

2. The Standard Optimal Growth Model

Let $\hat{K}(t) \geq 0$ and $\hat{C}(t) \geq 0$ represent the stock of capital and the level of consumption, at date *t*, in a given economy. These are the endogenous variables (state variable and control variable, respectively) of a typical optimal growth model. In this setting, output is generated through a production function exhibiting standard neoclassical properties (the function is continuous and differentiable, with positive first derivatives – positive marginal returns, and negative second derivatives – diminishing marginal returns). The arguments of the production function are capital and labour, and the function is presentable under the generic form

$$\hat{Y}(t) = F\left[\hat{K}(t), h(t)L(t)\right] \tag{1}$$

In equation (1), $\hat{Y}(t) \geq 0$ represents output; $L(t) \geq 0$ stands for the amount of labour (which, as a simplification, is assumed to coincide with the whole population of the economy); and $h(t) \geq 0$ is a measure of labour efficiency. Population and labour efficiency grow at constant rates n and g, respectively,

$$\dot{L}(t) = nL(t) \Rightarrow L(t) = L(0)e^{nt} \tag{2}$$

$$\dot{h}(t) = gh(t) \Rightarrow h(t) = h(0)e^{gt} \tag{3}$$

Output, capital, and consumption per efficiency unit of labour are defined in the following terms:

$$\hat{y}(t) \equiv \frac{\hat{Y}(t)}{h(t)L(t)}; \hat{k}(t) \equiv \frac{\hat{K}(t)}{h(t)L(t)}; \hat{c}(t) \equiv \frac{\hat{C}(t)}{h(t)L(t)} \tag{4}$$

Because the neoclassical production function (1) is homogeneous of degree 1, it can be presented in intensive form, i.e. taking variables per efficiency unit of labour,

$$\hat{y}(t) = f\left[\hat{k}(t)\right] \tag{5}$$

The capital accumulation constraint of the growth problem corresponds to the typical differential equation,

$$\dot{\hat{K}}(t) = F\left[\hat{K}(t), h(t)L(t)\right] - \hat{C}(t) - \delta\hat{K}(t),\ \hat{K}(0) \text{ given} \tag{6}$$

where $\delta \in (0,1)$ stands for the capital depreciation rate. Equation (6) can be written in intensive form, given the definitions in (4); the respective computation yields,

$$\dot{\hat{k}}(t) = f\left[\hat{k}(t)\right] - \hat{c}(t) - (n + g + \delta)\hat{k}(t),\ \hat{k}(0) \text{ given} \tag{7}$$

The representative agent maximises intertemporal utility, expressed under the form,

$$U(0) = \int_0^{+\infty} e^{-\rho t} u\left[\frac{\hat{C}(t)}{L(t)}\right] L(t) dt \tag{8}$$

In equation (8), $L(t)$ is the dimension of the household (in a representative agent model this coincides with the population), and $\frac{\hat{C}(t)}{L(t)}$ is per capita consumption, which in turn is the argument of the instantaneous utility function u. Parameter $\rho \geq 0$ represents the rate of time preference.

The instantaneous utility function is assumed to be a typical constant elasticity of intertemporal substitution utility function:

$$u\left[\frac{\hat{C}(t)}{L(t)}\right] = u[h(t)\hat{c}(t)] = \frac{[h(t)\hat{c}(t)]^{1-\theta}}{1-\theta},\ \theta \in (0, +\infty)\backslash\{1\} \tag{9}$$

Given the constant growth rates of population and labour efficiency, utility function (8) is representable as:

$$U(0) = h(0)^{1-\theta} L(0) \int_0^{+\infty} e^{-[\rho - n - (1-\theta)g]t} \frac{\hat{c}(t)^{1-\theta}}{1-\theta} dt \tag{10}$$

Condition $\rho - n - (1 - \theta)g > 0$ is a necessary condition for intertemporal utility to converge to a finite value.

The optimal growth problem consists in the maximisation of (10) subject to (7). Standard optimisation techniques allow for the straightforward derivation of an equation of motion for consumption. This is:

$$\dot{\hat{c}}(t) = \frac{1}{\theta}\left\{f'\left[\hat{k}(t)\right] - (\rho + \theta g + \delta)\right\}\hat{c}(t) \tag{11}$$

The growth model and the respective dynamics can be characterised taking into consideration the pair of differential equations (7) and (11). Results are well known: a unique steady state exists, and this equilibrium point is saddle-path stable.

To proceed with the characterisation of the model and the discussion of some relevant points, take a Cobb-Douglas production function, with $A \geq 0$ a productivity parameter and $\alpha \in (0,1)$ the output – capital elasticity,

$$\hat{Y}(t) = A\hat{K}(t)^{\alpha}[h(t)L(t)]^{1-\alpha} \Rightarrow \hat{y}(t) = A\hat{k}(t)^{\alpha} \tag{12}$$

For the Cobb-Douglas production function, it is straightforward to compute steady state levels of capital and consumption per efficiency unit of labour,

$$\hat{k}^* = \left(\frac{\alpha A}{\rho+\theta g+\delta}\right)^{1/(1-\alpha)} \tag{13}$$

$$\hat{c}^* = \left[1 - \frac{\alpha(n+g+\delta)}{\rho+\theta g+\delta}\right]\hat{y}^*, \text{ with } \hat{y}^* = A{\hat{k}^*}^{\alpha} \tag{14}$$

Equation (14) allows us to directly highlight the value of the savings rate in the steady state of the optimal growth model. This is:

$$\hat{\sigma}^* = \frac{\alpha(n+g+\delta)}{\rho+\theta g+\delta} \tag{15}$$

Note that condition σ 0*∈ (0,1) is satisfied under $\rho - n - (1-\theta)g > 0$.

In the optimal growth model, savings are endogenously determined given the optimal behaviour of the representative agent. This implies, necessarily, that dynamic inefficiency is not an issue in this setting. Dynamic inefficiency emerges when an increase in the savings rate leads to a fall in the steady state level of consumption, an event that can take place if, instead of an optimality framework, one considers a growth framework in which savings are exogenous.

Because the arguments regarding the effects of personality on consumption and savings choices and on growth involve taking an exogenous savings rate, in what follows we briefly approach the issue of dynamic inefficiency in a setting with an exogenous and constant savings rate. For an exogenous and constant savings rate, and a Cobb-Douglas production function, the steady state level of consumption would be (let σ be the constant savings rate):

$$\hat{c}^* = (1-\sigma)\sigma^{\alpha/(1-\alpha)}A^{1/(1-\alpha)}(n+g+\delta)^{-\alpha/(1-\alpha)} \tag{16}$$

The steady state consumption outcome is derived by applying condition $\dot{\hat{k}}(t) = 0$ to equation (7) under constraint $\hat{c}(t) = (1-\sigma)\hat{y}(t)$. Given (16), it is straightforward to compute the maximum level of consumption attainable in the steady state; this is known as the level of consumption respecting the golden rule. Below this level, the agent benefits from increasing savings because this leads to a rise in consumption. Above this maximum level, the issue of dynamic inefficiency emerges: higher savings will imply stronger capital accumulation but lower consumption in the long-term. The savings rate allowing for the satisfaction of the golden rule is:

$$\frac{\partial \hat{c}^*}{\partial s} = 0 \Rightarrow \frac{\alpha}{1-\alpha}(1-\sigma)\sigma^{\frac{\alpha}{1-\alpha}-1} = \sigma^{\frac{\alpha}{1-\alpha}} \Rightarrow \sigma = \alpha \tag{17}$$

For $\sigma = \alpha$, the steady state level of consumption attains its maximum value. Dynamic inefficiency implies $\sigma > \alpha$, while $\sigma < \alpha$ is a savings rate lower than the one that allows to maximize consumption.

By solving the optimal control problem, one computes (15) as the long-term savings rate associated with the maximisation of intertemporal consumption. Note that this savings rate does not correspond to the golden rule value. In fact, because under the model's assumptions, $\frac{n+g+\delta}{\rho+\theta g+\delta} < 1$, then $\hat{\sigma}^* < \alpha$: the savings rate that is derived endogenously from the intertemporal problem of utility maximisation conducts to a savings rate that is lower than the one allowing to maximise consumption in the long run. This is because the objective of the agent is not to maximise long term consumption but intertemporal utility and, given the existence of a positive discount rate, closer-in-time consumption is worth more than far-away-in-time consumption.

A particular version of the optimal growth model as characterised above is the endogenous growth version that emerges from assuming constant marginal returns from capital accumulation. Analytically, the transformation is simple; one just needs to impose the constraint $\alpha = 1$. In this case, there are no constant steady state levels of capital and consumption; these two variables will grow at a constant positive rate, which will be shaped by the various parameters of the model.

The growth rate of consumption in (11) becomes a constant growth rate over time,

$$\dot{\hat{c}}(t) = \frac{1}{\theta}[A - (\rho + \theta g + \delta)]\hat{c}(t) \tag{18}$$

This is the same growth rate at which capital will grow in the balanced growth path. Define ratio $\hat{\psi}(t) \equiv \hat{c}(t)/\hat{k}(t)$. For this ratio, given the equations of motion of consumption and capital, it is true that:

$$\dot{\hat{\psi}}(t) = \hat{\psi}(t) - \frac{\theta - 1}{\theta}(A - \delta) - \frac{1}{\theta}\rho + n \tag{19}$$

The long-term consumption-capital ratio is the constant value,

$$\hat{\psi}^* = \frac{\theta - 1}{\theta}(A - \delta) + \frac{1}{\theta}\rho - n \tag{20}$$

Observe that $\hat{c}^* = (1 - \hat{\sigma}^*)\hat{y}^* \Rightarrow \hat{\psi}^* = (1 - \hat{\sigma}^*)A \Rightarrow \hat{\sigma}^* = 1 - \frac{\hat{\psi}^*}{A}$: the steady state savings rate depends on the steady state consumption-capital ratio and on the level of productivity. This is in no way related with the savings rate derived in the neoclassical growth setting.

3. Non-optimal Savings and Neoclassical Growth

The main claim made in this paper is that agents know the optimal plan but do not necessarily follow it: there are psychological traits that see the individual deviate from pursuing optimal savings. Some individuals are spenders while others are savers, relatively to the optimal benchmark $\hat{\sigma}(t) = 1 - \frac{\hat{c}(t)}{f[\hat{k}(t)]}$. The representative agent in our model may deviate from the norm in either direction, to a greater or lesser extent.

Under the previous argument, the savings rate is generalised to:

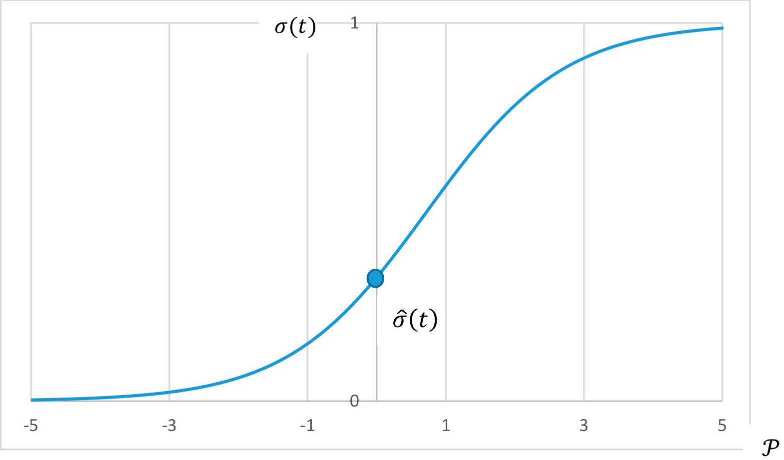
$$\sigma(t) = \frac{\{f[\hat{k}(t)] - \hat{c}(t)\}e^{\mathcal{P}}}{f[\hat{k}(t)]e^{\mathcal{P}} + \hat{c}(t)(1 - e^{\mathcal{P}})} \tag{21}$$

or, equivalently,

$$\sigma(t) = \frac{\hat{\sigma}(t)e^{\mathcal{P}}}{1 - \hat{\sigma}(t)(1 - e^{\mathcal{P}})} \tag{22}$$

In equations (21) and (22), $\mathcal{P} \in \mathbb{R}$ is a measure of personality. A neutral personality is such that $\mathcal{P} = 0$, in which $\sigma(t) = \hat{\sigma}(t)$ [with $\hat{\sigma}(t)$ the savings rate in the optimal problem]. A spender will be an individual for whom $\mathcal{P} > 0$, which implies $\sigma(t) < \hat{\sigma}(t)$; a saver will be the individual such that $\mathcal{P} > 0$ and, consequently, for whom, $\sigma(t) > \hat{\sigma}(t)$. Observe the extreme cases: if $\mathcal{P} \rightarrow -\infty$ then $\sigma(t) \rightarrow 0$; if $\mathcal{P} \rightarrow +\infty$ then $\sigma(t) \rightarrow 1$. For some $\hat{c}(t) < f[\hat{k}(t)]$, Figure 1 displays the relation between the personality measure and the savings rate that is implicit in equations (21) or (22),

Figure 1: The savings rate for different personalities



In the introduction, it was remarked that there is a fundamental distinction between the rate of time preference, ρ, and the measure of personality, $\mathcal{P}$. Although they are both essential in shaping consumption and savings trajectories, they play different roles in the model. The time preference translates the degree of impatience selected at the initial date to approach the optimal planning problem. The level of impatience attached to the subjective rate of time preference is consciously picked to attain a desired path for consumption that might privilege the short-run or, alternatively, the long-run. The personality parameter, in turn, is not a component of the optimal control problem. It is an uncontrollable force that is associated with the innate characteristics of the individual's personality. As mentioned in the introduction, the agent has little or no control over the features that define personality, as for example the traits of extraversion or neuroticism.

Therefore, differently from the discount rate, personality does not directly shape the intertemporal utility level. The impact of personalities that move away from the norm (i.e. from a neutral personality) is to make the agent deviate from approaching the optimal problem. The agent will know the problem and technically will have the ability and the means to solve it; however, the individual is endowed with an instinctive proclivity not to do it, given her underlying personality. If the agent has a 'spender' personality, she will fail to solve the optimal problem by default, i.e. she chooses a savings rate that is persistently lower than the benchmark. If the individual has a 'saver' personality, she will intuitively and systematically save more than the optimal problem would recommend. In this latter case, the agent fails to comply with utility maximisation by excess (i.e. by taking a savings rate higher than the optimal savings rate).

Equations (21) and (22), and Figure 1, characterise the aforementioned behavioural biases. They consider a benchmark case, of neutral personality, for which the savings rate is the one that coincides with the one underlying the optimality scenario. In this case, the constraint of the agent's problem is equation (7) and the model's solution is the one characterised in Section 2. As $\mathcal{P}$ deviates from the norm, assuming negative or positive values, equations (21) and (22) indicate that the savings rate chosen by the agent departs, for reasons of personality, from the savings rate under direct utility maximisation. It falls and approaches zero for progressively lower values of $\mathcal{P}$ when this is negative; and it increases and approaches 1 for progressively higher values of $\mathcal{P}$ when the parameter has a positive value. As it directly follows from equation (22), the actual savings rate is the optimal savings rate distorted by personality, and the stronger the personality (the higher the value of $\mathcal{P}$ in absolute value), the more $\sigma(t)$ deviates from $\hat{\sigma}(t)$.

As the analysis that follows will reveal, replacing $\hat{\sigma}(t)$ by $\sigma(t)$ in the growth setup will imply considering a dynamic equation for capital accumulation similar to the Solow growth model equation with exogenous and constant savings. The essential distinctive feature is that, in the personality framework, although exogenous, the savings rate is not constant. It is selected given the individual's personality, with reference to the optimal problem and, thus, with reference to the endogenous variables of the optimal problem, namely $\hat{k}(t)$ and $\hat{c}(t)$.

Because, in the personality setting, the savings rate is exogenous, the issue of dynamic inefficiency and of the determination of the golden rule becomes relevant. Deviations from the optimal savings rate, namely the possibility of a relatively high rate given a 'saver' personality profile, may push the economy into a dynamic inefficiency outcome, that one could never arise under optimality, regardless of the value of the subjective intertemporal discount rate.

As remarked above, the representative agent with a savings rate (21) or (22) will accumulate capital under a Solow-like equation, i.e.,

$$\dot{k}(t) = \sigma(t)f[k(t)] - (n + g + \delta)k(t) \tag{23}$$

The motion of variables $\hat{k}(t)$ and $\hat{c}(t)$, from which the value of $\sigma(t)$ directly depends, is given by equations (7) and (11). Under a Cobb-Douglas production function, the computation of the steady state capital stock yields the following expression, which should be compared with (13), i.e., with the long-term capital stock under utility maximisation (and neutral personality),

$$k^* = \left[\frac{\alpha A e^{\mathcal{P}}}{\rho + \theta g + \delta - \alpha(n + g + \delta)(1 - e^{\mathcal{P}})}\right]^{1/(1-\alpha)} \tag{24}$$

Observe that the derivative of the equilibrium level of capital with respect to personality is a positive value,

$$\frac{\partial k^*}{\partial \mathcal{P}} = \frac{1}{1-\alpha}\frac{\rho + \theta g + \delta - \alpha(n + g + \delta)}{\rho + \theta g + \delta - \alpha(n + g + \delta)(1 - e^{\mathcal{P}})}k^* > 0 \tag{25}$$

Therefore, the more $\mathcal{P}$ represents a 'saver' personality, the larger will be the stock of capital in the steady state: there is a positive relationship between $\mathcal{P}$ and capital accumulation, regardless of the values of the model's underlying parameters.

Consumption is defined by $c(t) = [1 - \sigma(t)]f[k(t)]$. The steady state value of consumption is, under the Cobb-Douglas technology,

$$c^* = \frac{\rho + \theta g + \delta - \alpha(n + g + \delta)}{\rho + \theta g + \delta - \alpha(n + g + \delta)(1 - e^{\mathcal{P}})}f(k^*) \tag{26}$$

with

$$f(k^*) = \left[\frac{\alpha e^{\mathcal{P}}}{\rho + \theta g + \delta - \alpha(n + g + \delta)(1 - e^{\mathcal{P}})}\right]^{\alpha/(1-\alpha)} A^{1/(1-\alpha)} \tag{27}$$

For $\mathcal{P} = 0$, steady state values (24) and (26) are identical to (13) and (14), as one would expect. Note as well that if $\mathcal{P} \to -\infty$ then $k^* \to 0$ and $c^* \to 0$ and if $\mathcal{P} \to +\infty$ then $k^* \to \left(\frac{A}{n+g+\delta}\right)^{1/(1-\alpha)}$ and $c^* \to 0$. The steady state amount of capital increases with a personality that is more amenable to save, as demonstrated through the computation of derivative (25). In turn, consumption will follow an inverted U-shape as the savings propensity increases: zero savings and total savings both lead to zero consumption in the long term. In this case, dynamic inefficiency emerges as a relevant issue. We will return to this after presenting explicitly the steady state savings rate in this case. Returning to (26) and rearranging:

$$c^* = \left[1 - \frac{\alpha(n+g+\delta)e^{\mathcal{P}}}{\rho+\theta g+\delta-\alpha(n+g+\delta)(1-e^{\mathcal{P}})}\right] f(k^*) \tag{28}$$

Therefore, the steady state savings rate is

$$\sigma^* = \frac{\alpha(n+g+\delta)e^{\mathcal{P}}}{\rho+\theta g+\delta-\alpha(n+g+\delta)(1-e^{\mathcal{P}})} \tag{29}$$

If $\mathcal{P} = 0$, the savings rate is $\sigma^* = \hat{\sigma}^*$, as presented in equation (15). If $\mathcal{P} \to -\infty$ then $\sigma^* = 0$ (no savings), and if $\mathcal{P} \to +\infty$ then $\sigma^* = 1$.

The equilibrium level of consumption per efficiency unit of labour can also be displayed as:

$$c^* = \frac{1-\sigma^*}{\sigma^*}(n+g+\delta)k^* = \frac{\rho+\theta g+\delta-\alpha(n+g+\delta)}{\alpha e^{\mathcal{P}}} k^* \tag{30}$$

Following (30), the ratio between consumption and capital in the steady state depends negatively on the personality parameter: the higher the value of $\mathcal{P}$, the lower is the consumption-capital ratio. Equation (30) returns the discussion to dynamic inefficiency and the golden rule. To proceed with this discussion, start by computing the derivative of steady state consumption with respect to the personality parameter:

$$\frac{\partial c^*}{\partial \mathcal{P}} = \frac{\rho+\theta g+\delta-\alpha(n+g+\delta)}{\alpha e^{\mathcal{P}}}\left(\frac{\partial k^*}{\partial \mathcal{P}} - k^*\right) \tag{31}$$

The golden rule condition is, then,

$$\frac{\partial k^*}{\partial \mathcal{P}} = k^* \Rightarrow \mathcal{P} = \ln\left[\frac{\rho+\theta g+\delta-\alpha(n+g+\delta)}{(1-\alpha)(n+g+\delta)}\right] \tag{32}$$

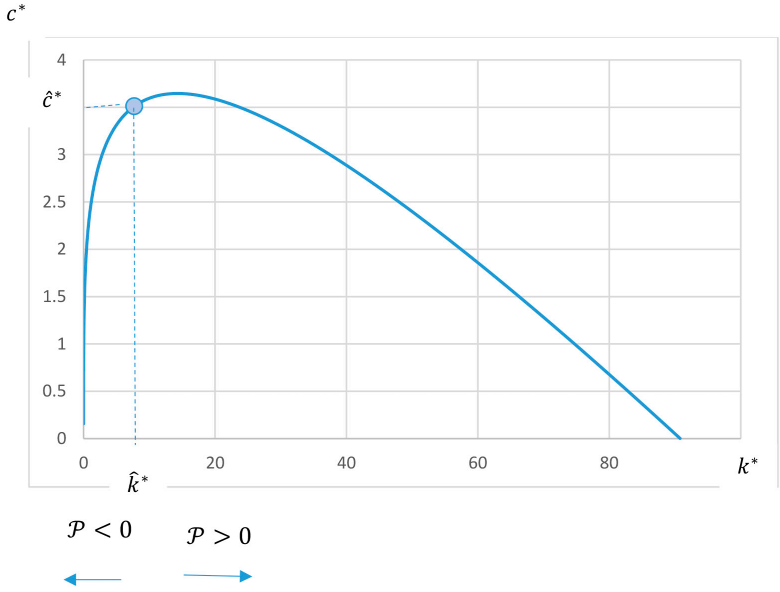
Recall that the golden rule implies that the level of consumption is maximised for a given savings rate. Substituting (32) into (29), one obtains a familiar outcome: $\sigma^* = \alpha$. This is the same result as in the neoclassical growth model with a fully exogenous savings rate. Further, observe that (32) is equivalent to:

$$\mathcal{P} = \ln\left[1 + \frac{\rho-n-(1-\theta)g}{(1-\alpha)(n+g+\delta)}\right] \tag{33}$$

Thus, it is not a neutral personality that maximises steady state consumption, but a 'saver' personality, given that under the imposed constraints $\mathcal{P} > 0$ in (33). The neutral personality allows for the agent to keep with the optimal plan, but it is not the personality leading to the golden rule result. The two coincide only when the extended discount rate, $\rho - n - (1 - \theta)g$, is hypothetically equal to zero. Dynamic inefficiency will emerge whenever $\mathcal{P}$ is larger than the expression in the right-hand-side of equation (33), that is, a 'saver' personality above this threshold is a personality leading to inefficient savings (to savings higher than those allowing for maximised steady state consumption).

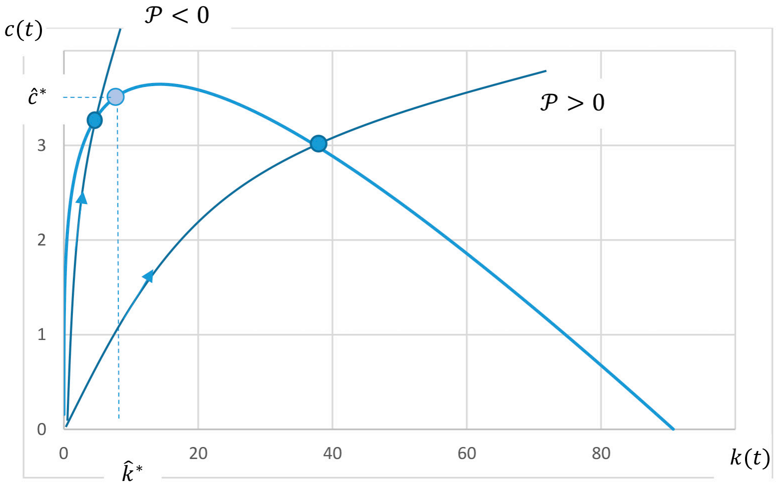
Next, we evaluate the location of the steady state point for different personalities, given the benchmark locus $(\hat{k}^*, \hat{c}^*)$. As mentioned, for $\mathcal{P} \to -\infty$, it is true that $k^* \to 0$ and $c^* \to 0$; on the other extreme, if $\mathcal{P} \to +\infty$ then $k^* \to \left(\frac{A}{n+g+\delta}\right)^{1/(1-\alpha)}$ and $c^* \to 0$. When $\mathcal{P}$ assumes the value in expression (33), the maximum point for c^* is accomplished. As remarked, this does not necessarily coincide with $\hat{c}^*$. The lack of coincidence between the optimal outcome and the golden rule result with exogenous savings, as highlighted before, arises because the objective of the agent is not to maximise long-term consumption; rather, to maximise intertemporal utility. Figure 2 represents the possible pairs (k^*, c^*) for different values of parameter $\mathcal{P}$. The inverted U-shaped relation between the values of the two variables is revealed.

Figure 2: Steady state pairs (k^*, c^*) for different personalities



Concerning transitional dynamics, note that the relation between $c(t)$ and $k(t)$ is concave and that, for a constant savings rate, the underlying dynamics in (23) are stable. Therefore, convergence to the equilibrium point is made through equation (23) as depicted in Figure 3 (the figure presents two examples of convergence to the steady state, for two different values of $\mathcal{P}$).

Figure 3: Transitional dynamics in the neoclassical model with non-optimal savings



4. Non-optimal Savings and Endogenous Growth

In the final part of Section 2, the endogenous growth version of the optimal growth model was characterised briefly. Exploring this version implies assuming a linear production function of the AK class, i.e., $f[\hat{k}(t)] = A\hat{k}(t)$. Under this technology of production, capital and consumption grow at a same steady state rate (which is the growth rate of consumption over the entire temporal horizon of the problem, i.e. the expression in equation (18)). Designate this growth rate by γ,

$$\gamma \equiv \frac{1}{\theta}[A - (\rho + \theta g + \delta)] \tag{34}$$

In the mentioned characterisation, the consumption-capital ratio, under the optimal problem, was defined by $\hat{\psi}(t)$, which allowed us to derive the corresponding balanced growth expression in (20).

As in the neoclassical growth case, the assumption of exogenous savings for personality traits other than those leading to the optimal result, are introduced via equations (21) or (22). Recovering the exogenous savings rate $\sigma(t)$, capital dynamics are now expressed under differential equation,

$$\dot{k}(t) = \sigma(t)Ak(t) - (n + g + \delta)k(t) \quad (35)$$

Consumption is defined as before, i.e., $c(t) = [1 - \sigma(t)]f[k(t)]$. Taking ratio $\psi(t) \equiv c(t)/k(t)$, observe that a straightforward equality emerges for the relation between consumption-capital ratios and savings rates,

$$\frac{\psi(t)}{\hat{\psi}(t)} = \frac{\sigma(t)}{\hat{\sigma}(t)e^{\mathcal{P}}} \quad (36)$$

The ratio between non-optimal and optimal consumption-capital ratios depends on the (non-optimal and optimal) savings rates, as well as the extent to which personality deviates from the established norm. Also observe that, in the current scenario, $\hat{\sigma}(t) = 1 - \frac{\hat{\psi}(t)}{A}$

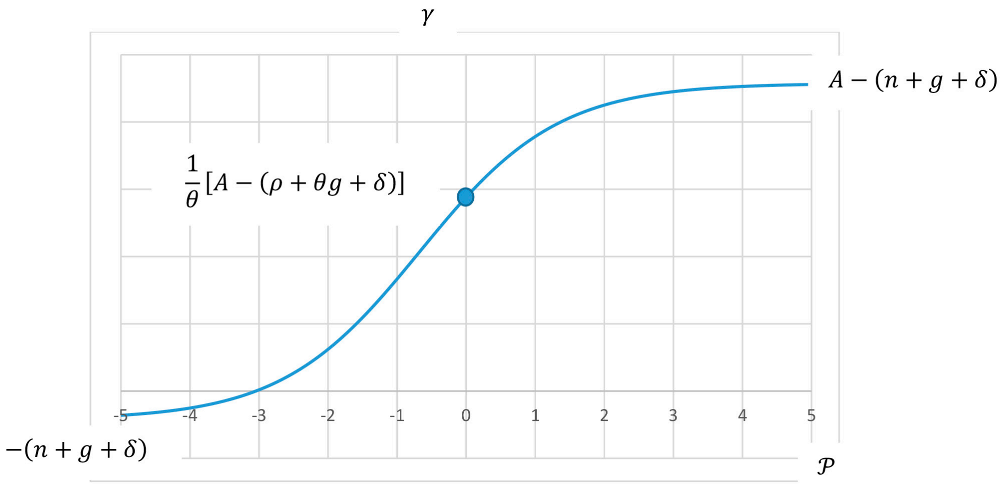
In the steady state, the consumption-capital ratio is equal to:

$$\psi^* = \frac{\sigma^*}{\hat{\sigma}^* e^{\mathcal{P}}}\hat{\psi}^* = \frac{A\hat{\psi}^*}{Ae^{\mathcal{P}} + \hat{\psi}^*(1 - e^{\mathcal{P}})} \quad (37)$$

Equation (37) indicates that if $\mathcal{P} = 0$ then $\psi^* = \hat{\psi}^*$; if $\mathcal{P} < 0$ then $\psi^* < \hat{\psi}^*$; and if $\mathcal{P} > 0$ then $\psi^* > \hat{\psi}^*$. Also note that, given (35), the steady state growth rate of capital (and also consumption) is, in the current setting,

$$\gamma = \sigma^* A - (n + g + \delta) = \frac{(A - \hat{\psi}^*)e^{\mathcal{P}}}{Ae^{\mathcal{P}} + \hat{\psi}^*(1 - e^{\mathcal{P}})}A - (n + g + \delta) \quad (38)$$

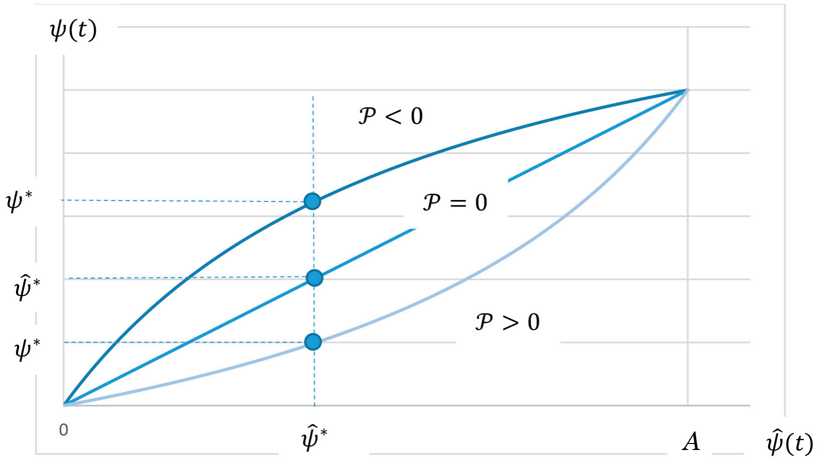
Figure 4: Steady state growth rate for different personalities (endogenous growth model)



If $\mathcal{P} = 0$, γ is the expression in equation (34). Figure 4 illustrates the value of growth rate γ for different values of $\mathcal{P}$ (note that if $\mathcal{P} \rightarrow -\infty$ then $\gamma \rightarrow -(n + g + \delta)$, and if $\mathcal{P} \rightarrow +\infty$ then $\gamma \rightarrow A-(n + g + \delta)$),

The relation between $\psi(t)$ and $\hat{\psi}(t)$, both in and out of the steady state, can also be depicted graphically. Figure 5 makes this representation for three different values of $\mathcal{P}$. They represent the stable paths in the direction of the steady state and the steady state points in each of the cases. Note that there are only two circumstances in which personality does not matter and where results are identical independently of the value of $\mathcal{P}$. These are the case in which consumption is zero and the case in which savings is zero (and consumption is equal to income). Figure 5 reveals that the more 'saver'/the less 'spender' an agent is, the lower will be the value of the steady state consumption-capital ratio.

Figure 5: Consumption-capital ratio (endogenous growth model)



5. Heterogeneity and Interdependence

Thus far, the analysis has assumed a decision-maker endowed with a potentially non-neutral personality. This implied the choice of a savings rate that is not necessarily the solution of the intertemporal utility maximisation planning problem and, as a result, the derived steady state and transitional dynamics do not coincide with the growth results that emerge from solving the optimal control problem. Notwithstanding this, the notion of a representative agent has not been abandoned, meaning that a single agent is responsible by addressing the planning problem, in the same way as in the standard optimal growth setting.

In this section, with the goal of introducing heterogeneity and interdependence in the analysis, the coexistence of two groups of agents is assumed. One of the groups will possess a neutral personality ($\mathcal{P} = 0$) and, thus, solves the maximisation problem, while the other group will be non-neutral regarding personality. This second group might be composed of savers ($\mathcal{P} > 0$) or spenders ($\mathcal{P} < 0$). The first group will be a share $\vartheta \in (0,1)$ of the population; the remaining agents, a percentage $1 - \vartheta$, will constitute the second group. The two groups contribute to the same production process, but with distinct savings rates.

To illustrate how heterogeneous agents coexist, recall the AK endogenous growth model in the previous section. In the endogenous growth scenario with heterogeneity, the capital accumulation equation takes the form:

$$\dot{k}(t) = [\vartheta\hat{\sigma}(t) + (1-\vartheta)\sigma(t)]Ak(t) - (n+g+\delta)k(t) \tag{39}$$

In the AK growth model with a single agent, the balanced growth consumption-capital ratio is given by expression (37) and the steady state growth rate is (38). To obtain these values for the new version of the model, begin by observing that the consumption-capital ratio is, in the new formulation of the model,

$$\psi(t) = \{1 - [\vartheta\hat{\sigma}(t) + (1-\vartheta)\sigma(t)]\}A \tag{40}$$

which is equivalent to:

$$\psi(t) = \frac{1 - \vartheta\hat{\sigma}(t)\left(1 - e^{\mathcal{P}}\right)}{1 - \hat{\sigma}(t)(1 - e^{\mathcal{P}})}\hat{\psi}(t) \tag{41}$$

Note that if $\vartheta = 0$, expression (41) is equivalent to (36). The steady-state consumption-capital ratio corresponds to:

$$\psi^* = \frac{A\left[1 - \vartheta\left(1 - e^{\mathcal{P}}\right)\right] + \vartheta\hat{\psi}^*\left(1 - e^{\mathcal{P}}\right)}{Ae^{\mathcal{P}} + \hat{\psi}^*(1 - e^{\mathcal{P}})}\hat{\psi}^* \tag{42}$$

If $\vartheta = 1$ then $\psi^* = \hat{\psi}^*$; and if $\vartheta = 0$ then the expression in (37) is obtained. Observe that under $\mathcal{P} > 0$, $\psi^* > \hat{\psi}^*$ and the distance between the two ratios falls with an increase in the value of ϑ. Under $\mathcal{P} < 0$, $\psi^* < \hat{\psi}^*$ and the distance between the two ratios also falls with an increase in the value of ϑ.

The next step in the analysis consists in computing the expression of the growth rate in the steady state. This is, in the current scenario,

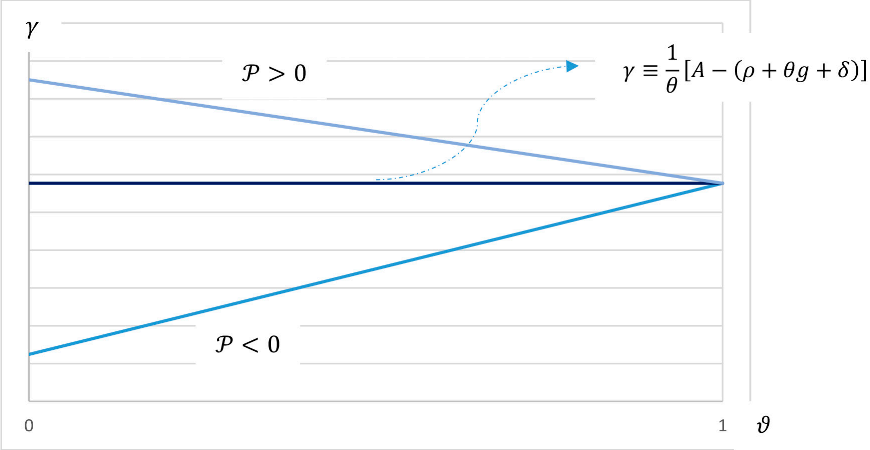
$$\gamma = [\vartheta\hat{\sigma}^* + (1-\vartheta)\sigma^*]A - (n+g+\delta) \tag{43}$$

Rearranging,

$$\gamma = \frac{\vartheta\left[Ae^{\mathcal{P}} + \hat{\psi}^*\left(1 - e^{\mathcal{P}}\right)\right] + (1-\vartheta)Ae^{\mathcal{P}}}{Ae^{\mathcal{P}} + \hat{\psi}^*(1 - e^{\mathcal{P}})}(A - \hat{\psi}^*) - (n+g+\delta) \tag{44}$$

Again, equality $\vartheta = 1$ brings us back to the optimal benchmark (i.e., equation (34)), while $\vartheta = 0$ implies the result in (38). Figure 6 draws growth rate (44) for different values of ϑ, assuming two distinct cases for personality levels $\mathcal{P} > 0$ and $\mathcal{P} < 0$. Results show how the optimal growth rate (regarding utility maximisation) is abandoned and replaced by a higher growth rate (case $\mathcal{P} > 0$) or by a lower growth rate ($\mathcal{P} < 0$), as the deviating personality gains relative weight (i.e., as the value of ϑ falls towards zero).

Figure 6: Balanced growth rate in the endogenous growth model, with two groups of agents



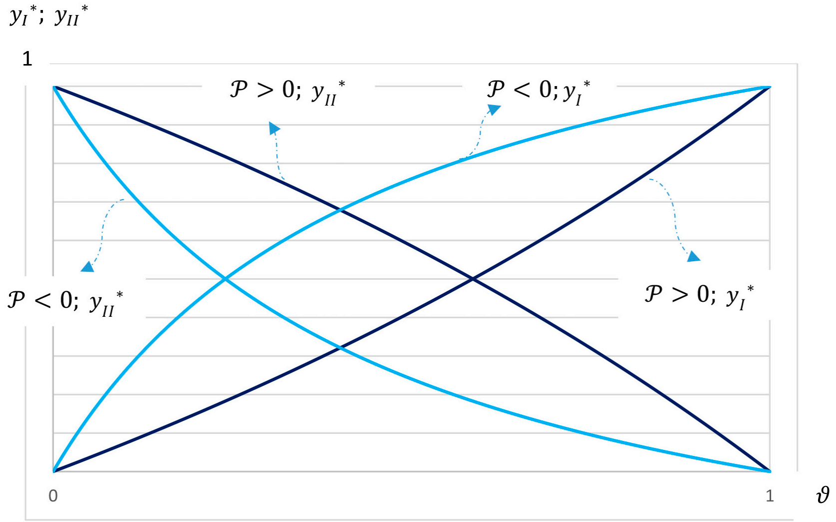
The analysis of the heterogeneity scenario is closed with the presentation of the shares of income of each group of agents, in the steady state. Observe that total income in the economy is $y(t) = [\vartheta\hat{\sigma}(t) + (1-\vartheta)\sigma(t)]Ak(t)$ and, therefore, the shares of income (ratios between the income of each group and total income) are, in the balanced growth path,

$$y_I^* = \frac{\vartheta\left[1 - \hat{\sigma}^*\left(1 - e^{\mathcal{P}}\right)\right]}{\vartheta[1 - \hat{\sigma}^*(1 - e^{\mathcal{P}})] + (1-\vartheta)e^{\mathcal{P}}} \tag{45}$$

$$y_{II}^* = \frac{(1-\vartheta)e^{\mathcal{P}}}{\vartheta[1 - \hat{\sigma}^*(1 - e^{\mathcal{P}})] + (1-\vartheta)e^{\mathcal{P}}} \tag{46}$$

Values y_I^* and y_{II}^* represent, respectively, the shares of income of the group with neutral personality and of the group for which $\mathcal{P} \neq 0$. Figure 7 displays the value of each share for $\vartheta \in (0,1)$ and taking into account two scenarios regarding personality (a positive $\mathcal{P}$ and a negative $\mathcal{P}$). The darker lines are drawn for $\mathcal{P} > 0$ and the lighter lines for $\mathcal{P} < 0$. In both cases, as ϑ increases, y_I^* increases and y_{II}^* falls. However, the most significant evidence drawn from the graphic is that y_I^* is larger for a 'spender' personality of the second group than for a 'saver' personality of this group. Accordingly, the second group always retains a larger income share if it is a group of 'savers' than a group of 'spenders'.

Figure 7: Steady state income shares for the two groups of agents, in the endogenous growth model



Although the above example has considered only two groups of agents endowed with potentially different personality indexes, the analysis could be extended to a wide variety of distinct personalities. In such case, the main conclusions would remain the same: heterogeneity influences aggregate results, concerning the growth of the economy and the value of the consumption-capital ratio and generates interdependence in the sense that the outcomes of 'savers' will depend on the quantity of 'spenders' in the economy and vice-versa.

6. Finite Life Cycles

In this section, the infinite horizon growth framework is replaced by a setting in which agents live for a period of time of length 1. Generations will not overlap; however, the following section will allow for a new generation to be born whenever an existing generation ceases to exist. Such a setting allows us to address the effects of personality on savings under an intergenerational perspective. This implies assuming a different kind of heterogeneity relative to the one discussed in Section 5. Now, agents may have different personalities, but they will be alive at non-overlapping periods of time.

Recall the solution of the optimal problem, in its endogenous growth version. This corresponds to the following set of differential equations:

$$\dot{\hat{k}}(t) = (\hat{\psi}^* + \gamma)\hat{k}(t) - \hat{c}(t) \tag{47}$$

$$\dot{\hat{c}}(t) = \gamma\hat{c}(t) \tag{48}$$

with $\hat{\psi}^*$ and γ given by the expressions in equations (20) and (34). It is straightforward to obtain the solution for the system of differential equations composed by (47) and (48):

$$\hat{k}(t) = \hat{k}(0)e^{(\hat{\psi}^*+\gamma)t} + \hat{c}(0)\frac{e^{\gamma t}}{\hat{\psi}^*}\left(1 - e^{\hat{\psi}^* t}\right) \tag{49}$$

$$\hat{c}(t) = \hat{c}(0)e^{\gamma t} \tag{50}$$

To establish a relation between the initial values of capital and consumption, one needs to know how the variables relate at the end date. Imagine that the transversality condition corresponds to zero savings at $t = 1$, $s(1) = 0$. In this case, $A\hat{k}(1) = \hat{c}(1)$; taking equations (49) and (50) into consideration, the condition is equivalent to:

$$A\hat{k}(0)e^{\hat{\psi}^*+\gamma} + A\hat{c}(0)\frac{e^{\gamma}}{\hat{\psi}^*}\left(1 - e^{\hat{\psi}^*}\right) = \hat{c}(0)e^{\gamma} \Leftrightarrow \hat{c}(0) = \frac{\hat{\psi}^* e^{\hat{\psi}^*}}{e^{\hat{\psi}^*} - \hat{\sigma}^*}\hat{k}(0) \tag{51}$$

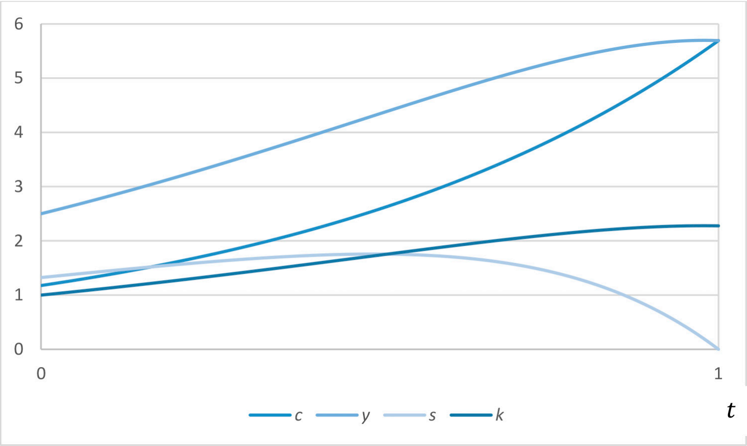
again with $\hat{\sigma}^* = 1 - \frac{\hat{\psi}^*}{A}$ the steady state savings rate of the optimal problem. Therefore, variables capital and consumption are presentable as:

$$\hat{k}(t) = \frac{e^{\hat{\psi}^*(1-t)} - \hat{\sigma}^*}{e^{\hat{\psi}^*} - \hat{\sigma}^*}e^{(\hat{\psi}^*+\gamma)t}\hat{k}(0) \tag{52}$$

$$\hat{c}(t) = \frac{\hat{\psi}^* e^{\hat{\psi}^*}}{e^{\hat{\psi}^*} - \hat{\sigma}^*}e^{\gamma t}\hat{k}(0) \tag{53}$$

Trajectories for endogenous variables (52) and (53), along with the trajectories for optimal income, $\hat{y}(t) = A\hat{k}(t)$ and savings, $\hat{s}(t) = \hat{y}(t) - \hat{c}(t)$, can be drawn for specific values of parameters. Let $A = 2.5$ $\rho = 0.05$, $n = 0.02$, $g = 0.04$, $\delta = 0.025$, $\theta = 1.5$. Figure 8 displays the four trajectories along the individual's life cycle. Savings fall to zero, according to the transversality condition, and consumption grows at a constant rate, and it equals income at the terminal date.

Figure 8: Life cycle trajectories of consumption, income, savings, and capital



Next, recall the non-optimality scenario, in which patterns of savings potentially vary with personality. In particular, consider capital accumulation equation (35) under savings rate (22). The solution of the system composed by the equations of motion for $k(t)$, $\hat{k}(t)$ and $\hat{c}(t)$ delivers the earlier trajectories for the optimal variables, (52) and (53), plus the following equation for the motion of $k(t)$,

$$k(t) = \left[\frac{e^{\widehat{\Psi}^*(1-t)} - \sigma^*}{e^{\widehat{\Psi}^*} - \sigma^*}\right]^{\frac{\sigma^*}{\hat{\sigma}^* e^{\mathcal{P}}}} e^{(\widehat{\Psi}^*+\gamma)t}\hat{k}(0) \tag{54}$$

The result for consumption emerges from the corresponding definition, i.e., $c(t) = [1 - \sigma(t)]Ak(t)$. Note that, given the definition of $\sigma(t)$ and expressions (52) and (53), it is possible to derive an explicit formula for the optimal savings rate as a function of time:

$$\hat{\sigma}(t) = \hat{\sigma}^* \frac{e^{\widehat{\Psi}^*(1-t)} - 1}{e^{\widehat{\Psi}^*(1-t)} - \hat{\sigma}^*} \tag{55}$$

Replacing (55) into (22),

$$\sigma(t) = \frac{\hat{\sigma}^*\left[e^{\widehat{\Psi}^*(1-t)} - 1\right]e^{\mathcal{P}}}{(1 - \hat{\sigma}^*)e^{\widehat{\Psi}^*(1-t)} + \hat{\sigma}^*\left[e^{\widehat{\Psi}^*(1-t)} - 1\right]e^{\mathcal{P}}} \tag{56}$$

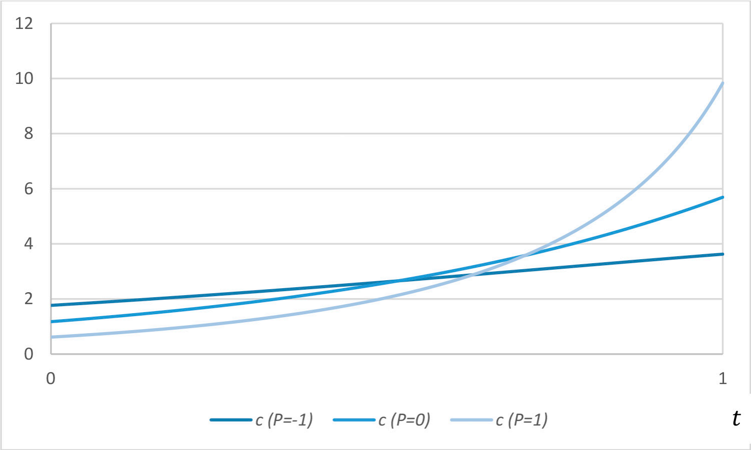
Putting all the above information together, consumption as a function of time is derived:

$$c(t)=A\left[\frac{(1-\hat{\sigma}^*)e^{\hat{\Psi}^*(1-t)}}{(1-\hat{\sigma}^*)e^{\hat{\Psi}^*(1-t)}+\hat{\sigma}^*\left[e^{\hat{\Psi}^*(1-t)}-1\right]e^{\mathcal{P}}}\right]\left[\frac{e^{\hat{\Psi}^*(1-t)}-\sigma^*}{e^{\hat{\Psi}^*}-\sigma^*}\right]^{\frac{\sigma^*}{\hat{\sigma}^*e^{\mathcal{P}}}}e^{(\hat{\Psi}^*+\gamma)t}\hat{k}(0) \quad (57)$$

Observe that for $\mathcal{P}$ = 0, expressions (54), (56) and (57) have correspondence with the optimal benchmark results. The new equations are generalisations for possible different savings behaviours (for positive or negative values of $\mathcal{P}$).

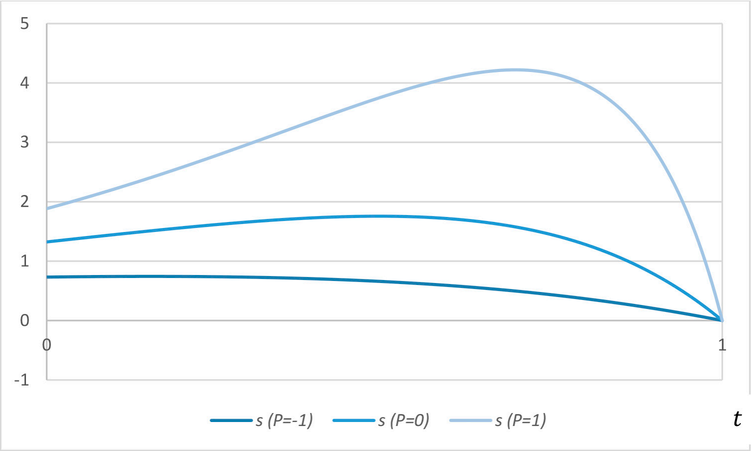
To understand what happens to consumption and savings when personalities differ, the parameter values already used to draw Figure 8 are recovered and three personality scenarios are assumed: $\mathcal{P}$ = –1, $\mathcal{P}$ = 0, $\mathcal{P}$ = 1. Recall that the first case defines a spender and the last one a saver; in all cases, transversality condition $s(1)$ = 0 is maintained. Figure 9 presents the trajectories for consumption, while Figure 10 displays the trajectories for savings.

Figure 9: Life cycle trajectory of consumption for different personalities of the representative agent



The graphics reveal that a 'saver' will save more along the life cycle than a 'spender'; however, to obey the transversality condition, there is a sharper decline in the savings of the 'saver' near the end of the life cycle. This implies, as Figure 9 corroborates, that consumption of this class of agents remains relatively low along a large part of the life cycle but it increases sharply in the final stage of the agent's life.

Figure 10: Life cycle trajectory of savings for different personalities of the representative agent



7. Saving for Future Generations

In this section, a similar framework to the one already explored is adopted, but with a few key changes. In particular, consider the following assumptions: (*i*) Personality no longer provokes a deviation from optimal behaviour: the representative agent solves the optimal problem; (*ii*) Now, personality has influence on intergenerational altruism, i.e. on the amount of savings the agent is willing, in the last moment of life, to transfer to the next generation; (*iii*) A sequence of different generations is considered. These do not overlap but they are adjacent. In the moment one generation abandons the economy, another generation enters the economy; (*iv*) The capital accumulated by one generation is not transmissible to the next generation (e.g. it might be interpreted as a form of embodied human capital); (*v*) The initial level of capital of a new generation is equal to $\hat{k}(0)$ plus savings transferred from the previous generation.

Let $\hat{\sigma}(1) \in (0,1)$ be the savings rate at the terminal date for a neutral personality ($\mathcal{P} = 0$); a saver concerned with the next generation will adopt a savings rate higher than $\hat{\sigma}(1)$, and the opposite will occur for a spender. A usual S-shaped function of $\mathcal{P}$ is, then, adopted for the savings rate at the terminal date,

$$\sigma(1) = \frac{\hat{\sigma}(1)e^{\mathcal{P}}}{1 - \hat{\sigma}(1)(1 - e^{\mathcal{P}})} \tag{58}$$

According to expression (58), a fully egoistic agent will save nothing for the generation that follows ($\mathcal{P} \to -\infty$), while a fully altruistic agent will save the whole income received in the final date ($\mathcal{P} \to +\infty$).

The main change relative to the model discussed in the previous section concerns the transversality condition, which is now:

$$\hat{s}(1) = \sigma(1)\hat{y}(1) \Leftrightarrow A\hat{k}(1)[1-\sigma(1)] = \hat{c}(1) \tag{59}$$

Taking into consideration equations (49) and (50), and proceeding as in (51), for the new transversality condition the following generalisation of result (51) is obtained:

$$\hat{c}(0) = \frac{\hat{\psi}^* e^{\hat{\psi}^*}[1-\sigma(1)]}{e^{\hat{\psi}^*} - \hat{\sigma}^* - \left(e^{\hat{\psi}^*}-1\right)\sigma(1)}\hat{k}(0) \tag{60}$$

Under (60), the capital and consumption expressions are:

$$\hat{k}(t) = \frac{e^{\hat{\psi}^*(1-t)} - \hat{\sigma}^* - \left[e^{\hat{\psi}^*(1-t)}-1\right]\sigma(1)}{e^{\hat{\psi}^*} - \hat{\sigma}^* - \left(e^{\hat{\psi}^*}-1\right)\sigma(1)} e^{(\hat{\psi}^*+\gamma)t}\hat{k}(0) \tag{61}$$

$$\hat{c}(t) = \frac{\hat{\psi}^* e^{\hat{\psi}^*}[1-\sigma(1)]}{e^{\hat{\psi}^*} - \hat{\sigma}^* - \left(e^{\hat{\psi}^*}-1\right)\sigma(1)} e^{\gamma t}\hat{k}(0) \tag{62}$$

Taking the same parameter values as in the already illustrated examples, Figure 11 and Figure 12 display the trajectories of capital and consumption for the extreme cases $\sigma(1) = 0$ and $\sigma(1) = 1$, as well as an intermediate case, $\sigma(1) = 0.5$. The graphics reveal a stronger capital accumulation for higher terminal savings and the opposite behaviour for consumption (in the extreme case $\sigma(1) = 1$, consumption is equal to zero throughout the life cycle).

Figure 11: Capital trajectories for different transversality conditions

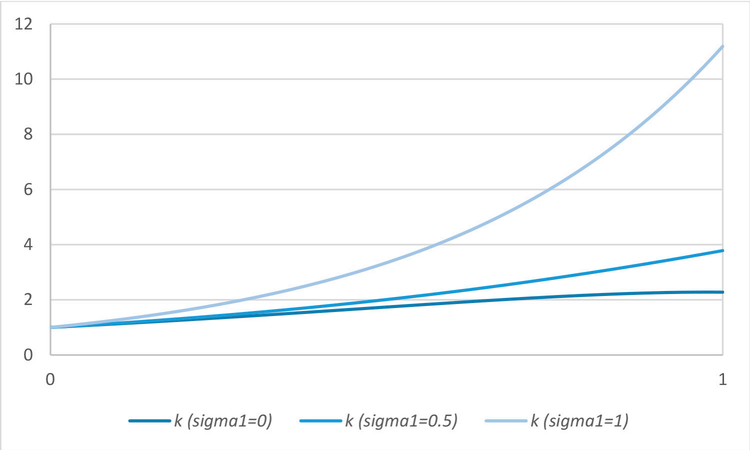
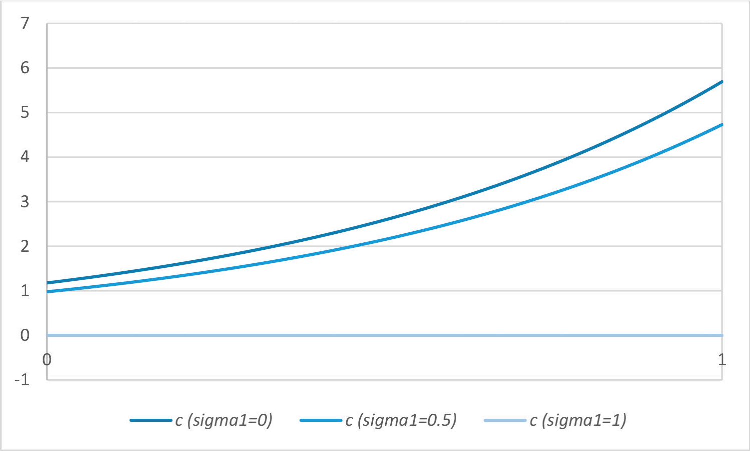
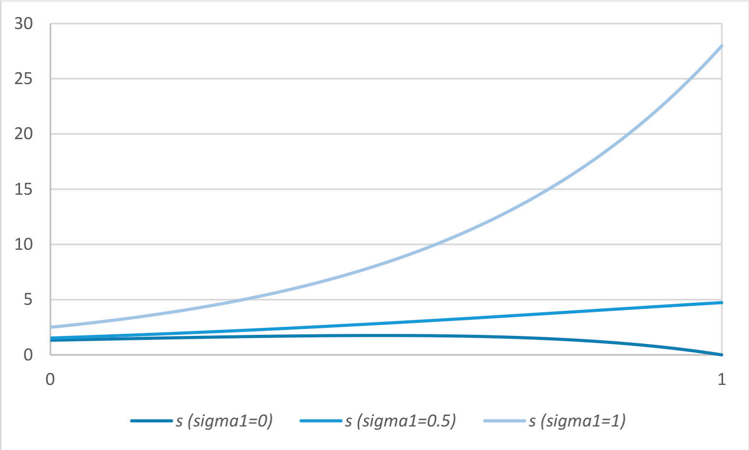


Figure 12: Consumption trajectories for different transversality conditions



One can also display the trajectories for savings under the same terminal savings assumptions. This is done through Figure 13. Different trajectories of savings emerge for different personalities associated with intergenerational altruism.

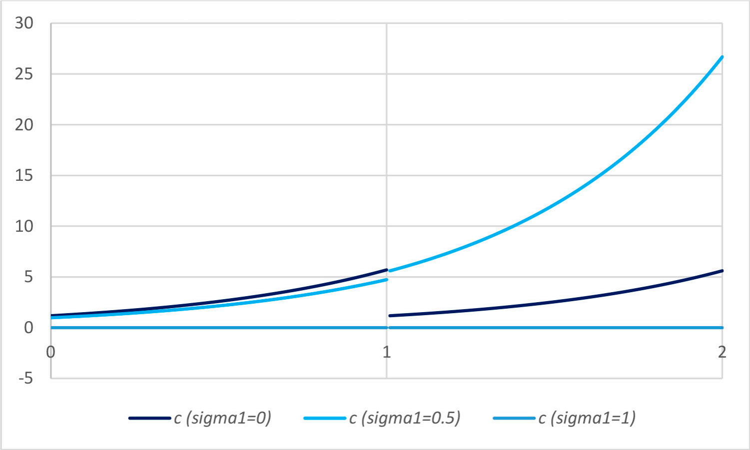
Figure 13: Savings trajectories for different transversality conditions



Now, it is possible to connect generations by establishing that the new generation starts with a capital endowment equal to $\hat{k}(0)_{[n]} = \hat{k}(0)_{[o]} + \sigma(1)_{[o]}\hat{y}(0)_{[o]}$, where the subscripts [*n*] and [*o*] refer to the new and the old generations. The trajectories of consumption of two consecutive generations are presented for the three cases mentioned above (Figure 14). Note that in the extreme case of saving all the resources for the future, no consumption takes place (neither in the current generation nor in the next). The other extreme, of no concern with future generations, implies that the future generation starts at the same point as the previous one, and therefore it mimics exactly the same consumption path (with no growth relative to the previous generation).

In the intermediate case, there is growth: the second generation starts a little bit above the terminal point of the first, leading to a long run consumption performance that is notoriously preferable. Therefore, one concludes that in this interpretation a kind of golden rule solution holds: saving too much or too little to the next generation is not optimal for any of the generations. The optimal result will be found somewhere in the middle, depending always on how much the current generation is willing to put aside in favour of the generation that follows.

Figure 14: Consumption trajectories for two consecutive generations



8. Conclusions

Optimal growth models are designed to explain consumption choices over time. These choices are made by a representative agent who selects, at every instant, how much to consume and how much to save. Agents with identical time preference, identical initial endowments of capital, and identical levels of productivity should, therefore, make the exact same choices and their behaviour may be reduced to the behaviour of a representative agent. In reality, even when individuals face no obstacles in knowing and understanding the optimal plan, they may choose not to follow it and, according with their personality, they may adopt a posture of 'saver' or 'spender'.

This study has analysed the effects of such postures in the context of standard neoclassical and endogenous growth models. Deviating from optimal behaviour introduces distortions in steady state results and on transitional dynamics. Although knowledgeable of the optimal problem, instead of solving it, the agent selects an exogenous savings rate (although this is contingent on the evolution of the optimal values of capital and consumption). In such a scenario, a Solow-like equation of capital accumulation is recovered as the central analytical piece to explain growth. The analytical inspection of the model has, then, allowed us to explore important new insights about the mechanics of growth, under a representative agent setting, but also assuming heterogeneity and interdependence.

Savings behaviour was also analysed in the context of a finite life cycle model. This model clarifies further the implications of adopting the 'saver' or 'spender' profile. Two different approaches were followed: first, it was assumed that only one generation populates the economy, and therefore the agent has, in any case, and regardless of her personality, no interest in saving beyond the end of the life cycle. In this case, 'savers' will be individuals with low levels of consumption along a large portion of the life cycle, but who increase their consumption sharply at the end of their lives. 'Spenders' will adopt the opposite behaviour: they will consume more than the optimal level in the early stage of life, and this must be compensated for in later stages with lower consumption.

In the second approach, every agent solves the same optimal problem but chooses a different transversality condition for savings. This might be interpreted as distinct propensities for altruism. The altruistic individual will select a relatively high savings rate at the terminal date, while the egoistic agent will opt for a relatively low savings rate at the end date. The savings that are accumulated in this way will serve to enhance the consumption opportunities of the generation that follows, becoming thus a fundamental driver of future growth (higher or lower, depending on the personality of the agent in the current generation, and therefore of her propensity to save for future generations).

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Endnote

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