

Information Flow Interpretation of Heteroskedasticity for Capital Asset Pricing: An Expectation-based View of Risk

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ABSTRACT

The Heteroskedastic Mixture Model (HMM) of Lamoureux, and Lastrapes (1990) is extended, relaxing the restriction imposed on the mean i.e. $\mu_{t-1}=0$. Instead, an exogenous variable r_m , along with its vector β_m , that predicts return r_t is introduced to examine the hypothesis that the volume is a measure of speed of evolution in the price change process in capital asset pricing. The empirical findings are documented for the hypothesis that ARCH is a manifestation of time dependence in the rate of information arrival, in line with the observations of Lamoureux, and Lastrapes (1990). The linkage between this time dependence and the expectations of market participants is investigated and the symmetric behavioural response is documented. Accordingly, the tendency of revision of expectation in the presence of new information flow whose frequency as measured by 'volume clock' is observed. In the absence of new information arrival at the market, investors tend to follow the market on average. When new information is available, the expectations of investors are revised in the same direction as a symmetric response to the flow of new information arrival at the market.

1. INTRODUCTION

THE AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (ARCH) model of Engle (1982) provides better forecasts of volatility, relying on two phenomena of financial market volatility; volatility clustering and mean reversion in volatility. Also, volatility clustering (Mandelbrot 1963) suggests that it is more informative to account for more recent innovations in return than older ones. Thus, ARCH assigns higher weight to the more recent return innovations. ARCH models are backed up by theories such as the observation of volatility clustering phenomenon (Mandelbrot 1963) and the subsequent conceptualisation of the Efficient Market Hypothesis² (Fama 1965).

The Mixture of Distribution Hypothesis (MDH) as examined by Clark (1973) and Tauchen and Pitts (1983), finds that the evolution of returns and the trading volume are driven by the same latent mixing variable, which reflects the amount of new information flows into the market. Thus ARCH should accept the hypothesis that daily price changes and stock volume are mixtures of independent normals with the same mixing variable, as originally examined by Clark (1973). Nowadays the trading mechanism is highly technological, involving online real-time trading platforms and incredibly fast flows of information as a result of advances in information technology. The origin of the information flows into the market is witnessed by the traders who pick and dispose of stocks based on the information available to them. In a typical order book, traders enter the quantity of stock and the price of the order before the execution of transactions. Upon execution of trades, first any shock to the return volatility is reflected in the stock volume and then the stock price changes are generated accordingly. Therefore the asset pricing models should qualify for mixed distribution properties.³

Non-availability of a numerical measure for information flows into the market has prevented scholars from providing reliable evidence on the conclusion of the thesis of Clark (1973). In the main, two competing random variables, volume and the number of transactions, as proxies for the rate of information arrival at the market, have been tested. Harris (1987) suggests the daily number of transactions may be a useful proxy instrument in accounting for the rate of information arrival, under the assumption that transactions occur at a uniform rate in event time. However, volume claims priority and has proven to be the most promising candidate for the mixing variable (See e.g. Clark 1973; Epps and Epps 1976; Tauchen and Pitts 1983; Ross 1987; Andersen 1996) as it accounts for both the rate (by frequency) and the amount (by scale) of information arrival at the market.

A considerable amount of effort has been devoted in the literature to examining the validity of the hypothesis that daily price changes and volumes are driven by the same mixing variable, which is identified as the directing random variable in the information arrival process (i.e. the directing process). Notably, a lack of parsimony and extensive theoretical rigour were observed in many studies conducted in this regard. Thus, citations of the valuable literature have been limited to a handful of scholarly work. However, Lamoureux, and Lastrapes (1990) postulate the validity of the mixture model in the presence of heteroscedasticity more precisely and parsimoniously using a plain vanilla Generalised ARCH (1,1) process, and demonstrate the ARCH effect vanishes when volume is included in the conditional variance equation. This implies that volume and daily price changes are driven by the same latent mixing variable. Thus, the form of evidence supports empirically the hypothesis that ARCH is a manifestation of time dependence in the rate of information arrival for individual stocks.

Lamoureux, and Lastrapes (1990) constrain conditional mean of return (r_t) in the mean equation to zero (Lamoureux, and Lastrapes 1990 p

222). Thus, the return is simply the contemporaneous surprises. This restricts the avenue of documenting the linkage between the ARCH effect and the conditional expectation which, in turn, will provide much needed evidence on the hypothesis that trading volume is a measure of speed of evolution⁴ in the price change process that ARCH should be capable of accounting for.⁵ Clark (1973) and many others demonstrate that trading volume is positively and serially correlated with squared price changes (r_t^2) and find that trading volume measures the speed of evolution in the price change process. The expectations of investors about future stock returns are also determined by the arrival of 'new' information at the market (See especially Lambert and Verrecchia 2010). Therefore the revision in the expectation as measured by the covariance (in mean) might provide much useful evidence as to the validity of the Heteroskedastic Mixture Model (HMM) in capital asset return modeling (especially in capital asset pricing).

The objective of this paper is to examine the validity of the hypothesis that ARCH is a manifestation of time dependence in the rate of information arrival at the market in conjunction with capital asset pricing. The way to examine this hypothesis under HMM is to document whether the trading volume is a measure of speed of evolution in the price change process in capital asset pricing. The Heteroskedastic Mixture Model is redesigned to examine the implications of the latent mixing variable on expectation, while also testing the hypothesis that the heteroscedasticity of variance of daily price increments is positively related to the directing variable. If the model under this framework is valid, a symmetric response⁶ is expected from market participants, which may be reflected in the covariance between market return and daily equilibrium price changes. A successful capital asset pricing model incorporates the flow of new information arrival at the market for precious estimate of coefficients.

This article is structured as follows. Section 2 provides a theoretical framework for the extension of HMM. Section 3 describes the methodological approach followed, and outlines the limitations. Empirical findings are summarised in section 4, along with theoretical explanations for the underlying arguments. Section 5 provides concluding remarks.

2. THE CONCEPTUAL FRAMEWORK

2.1 *The Heteroskedastic Mixture Model for capital asset pricing*

The Generalised ARCH or GARCH model of Bollerslev (1986) for capital asset pricing is given by:

$$r_{it} = \beta_0 + \beta_m r_{mt} + \varepsilon_t, \quad (1)$$

$$\varepsilon_t \setminus (\varepsilon_{t-1}, \varepsilon_{t-2}, \dots) \sim N(0, h_t) \quad (2)$$

$$h_t = \alpha + \pi(L)\varepsilon_{t-1}^2 + \lambda(L)h_{t-1} \quad (3)$$

Where r_{it} is the return of stock i at time t , β_m is the beta coefficient, and β_0 is the intercept term, ε_t is the error term at time t , r_{mt} is the market return⁷ at time t . L is the lag operator, and $\alpha > 0$. π and λ are the ARCH and GARCH coefficients respectively, which should theoretically be positive in order for shocks to the volatility to persist over time.

Now, let δ_{jt} denote the j^{th} intraday equilibrium price increment in day t that is constructed in the sense of Lamoureux and Lastrapes (1990). This implies:

$$\varepsilon_t = \sum_{j=1}^{n_t} \delta_{jt} \quad (4)$$

Where n_t is the random mixing variable that represents the stochastic rate at which the information flows into the market; ε_t is subordinated to δ_j following Mandelbrot and Taylor (1967), Clark (1973), Westerfield (1977), Harris (1987), and Lamoureux and Lastrapes (1990). It is noted that ε_t is drawn from a mixture of distributions⁸ where the variance of each distribution depends upon the information arrival time. If δ_{jt} is i.i.d.⁹ with mean zero and variance σ^2 and n_t (the directing variable) is sufficiently large, then ε_t follows a normal distribution as $\varepsilon_t | n_t \sim N(0, \sigma^2 n_t)$. However, if n_t varies over time, the Central Limit Theorem (CLT) does not apply, as it holds only when the stochastic variable n_t (i.e. the rate at which information arrives at the market) is constant (See e.g. Lamoureux and Lastrapes 1990 p 222). This leads to rejection of the normality assumption in the unconditional distribution even if CLT applies, when the variation in n_t occurs over time. It is, however, assumed that equilibrium price increments are conditionally normally distributed and CLT can be invoked.

Following Lamoureux and Lastrapes (1990), assume a daily information arrival at the market, n_t that is serially correlated which could be expressed in the form:

$$n_t = k + b(L)n_{t-1} + u_t \quad (5)$$

where k is a constant, $b(L)$ is the lag operator of order q , and u_t is the *white noise*. Since n_t is not observable, a proxy is used for daily number of information arrival which is the stock volume as mentioned. If n_t is serially correlated, volatility and trading volume will also be jointly serially correlated. Therefore trading volume is useful in providing evidence on the behaviour of the second order moments of returns (See Bollerslev *et al* 1994).

As Lamoureux and Lastrapes (1990) note, innovations to the mixing variable persist according to the autoregressive structure of $b(L)$. It is duly noted that obviously k and b are non-negative. Such an autoregressive structure has the property of capturing the information innovations in the infor-

mation arrival process that should theoretically be equal or empirically approximately equal to the information innovations in the equilibrium price change process, if MDH can be invoked. Clark (1973), Epps and Epps (1976), Tauchen and Pitts (1983), and Harris (1987) document that information arrival at the market (the mixing variable) causes a joint return volatility-volume relation. This study does not attempt to decompose the content of the information (for example quality, precision, accuracy etc.) as they are reflected in the price changes as and when the investors judge the relevant phenomenon (See e.g. Fama 1965).¹⁰

Define $\zeta = E(\varepsilon_t^2 | n_t)$. If the mixture model is valid, then $\zeta = \sigma^2 n_t$ in the sense of Lamoureux and Lastrapes (1990) and Clark (1973 p 140). As such, substituting the moving average representation of (5) would results in equation (6) which is expressed as:

$$\zeta_t = \sigma^2 k + b(L)\zeta_{t-1} + \sigma^2 u_t \quad (6)$$

In the sense of Lamoureux and Lastrapes (1990), equation (6) captures the type of persistence in the conditional variance that may be estimated by GARCH models in which innovations to the information arrival process lead to momentum in the squared surprises of daily stock returns. Autoregressive-Moving Average errors yield more efficiently than Ordinary Least Squares (OLS) errors that could be estimated by equation (6) when data exhibit serial correlation (see, for example, Fang and Koreisha 2004).

2.2 The Capital Asset Pricing Model and the covariance

The use of a simple AR (1) process to account for innovations in the information arrival process (a stochastic process with positive increments) as in equation (5) makes the estimation process more efficient and superior than Ordinary Least Squares estimates. Adrian and Franzoni (2009) demonstrate that the traditional OLS regression approach ignores investors' gains from previous errors and, as such, the CAPM is often rejected. This motivates the use of ARCH structures in capital asset pricing and autoregressive structures for accounting innovations in the information arrival process. Further, scholars argue that investors engage in a learning process in assessing long run beta. Taking the learning process into account, scholars demonstrate that the evolution of factor loading¹¹ follows an AR (1) process to capture the momentum in the learning process.

The Capital Asset Pricing Model of Sharpe (1964) sets the benchmark for the asset pricing literature. Such a model used under GARCH or Generalised ARCH for stock return modelling is given in equation (1) above. The return estimation equation suppressing the intercept term could be written as:

$$r_{it} = \beta_m r_{mt} + \varepsilon_t \quad (7)$$

β_m could therefore be represented as in the equation (8) below,

$$\beta_m = \frac{\text{Covariance}(R_i, R_m)}{\text{Variance}R_m} \quad (8)$$

Equation (7) is now rearranged in the following manner, decomposing the beta coefficient,

$$r_{it} = \left(\frac{\text{Covariance}(R_i, R_m)}{\text{Variance}(R_m)} \right) r_{mt} + \varepsilon_t \quad (9)$$

Measuring the effect of the information on expected returns through covariance is not new in the asset pricing literature. Lambert and Verrecchia (2010 p 1) point out a valid proposition, that the only way information can affect cost of capital is through its impact on the covariance of the firm's cash flows with the market. Covariance between r_m at time t and r_i at time t is now written rearranging other variables in the following manner:

$$\text{Covariance}(R_{it}, R_{mt}) = \left(\frac{r_{it} - \varepsilon_t}{r_{mt}} \right) \sigma_{mt}^2 \quad (10)$$

2.3 The white noise (u_t) and the error term (ε_t)

Suppressing constant k , equation (5) can be rearranged as:

$$u_t = n_t - b(L)n_{t-1} \quad (11)$$

In Equation (11) u_t is simply the innovation in the information arrival process as proxied by the stock volume.

In an OLS linear regression relationship (as in mean), u_t is not simply equal to ε_t because Engel's (1982) conceptualisation of information innovation in returns persists, according to the autoregressive structure of ε_t^2 . If HMM is true, given the nature of the trading mechanism, u_t should also play the role of aggregating contemporaneous surprises of each trade (j^{th} trade) in day t as ε_t in the mean equation (1) within the mixture of distribution framework (see equation 4 and endnote 8). Thus, to motivate the conceptualisation of this study, innovations to the mixing variable in the information arrival process as proxied by stock volume can be included in the information set in the mean representation (See e.g. Bauer and Nieuwland 1995 p.140 and Clark 1973 p.139 for similar arguments).

McNees (1980) points out that large and small errors tend to cluster together. This implies large errors are followed by large errors

and vice versa. Accordingly, if ε_{t-1}^2 is large, ε_t^2 will also be large and vice versa. Ying (1966 p 683) demonstrates that if the log volumes are large, it is expected that log prices will also be large, and vice versa. This implies that the volatility clustering of Mandelbrot (1963) should exist between volume and stock returns. Clark (1973 p 142) documents that if the trading volume is the directing process, the relationship between trading volume and the equilibrium price change variance, r_{it}^2 (squared price changes) should be linear, with the proportionality coefficient representing the variance of equilibrium price changes. Engle *et al* (1987) parameterise the conditional variance as a function of the information set available to investors and assume that the most useful information available to agents is the previous innovations or surprises (i.e. ε_t in the mean equation). Such postulation states that $\text{Var}(\varepsilon_t \setminus \text{all available information}) = h_t$. Standard deviation $\sqrt{\zeta_t}$ or $\sqrt{h_t}$, as the case may be, is included in the determination of expectation (i.e. $\text{Covariance}(R_{it}, R_{mt})$ in the coefficient estimate in the mean equation).

A revision in the direction of the covariance between market return and stock return in the mean equation (1) should occur when persistence of the ARCH effect is almost neutralised (it becomes negligible) by the inclusion of stock volume in the conditional variance equation. When the ARCH effect becomes insignificant, persistence of variance capturing the time dependence in the new information arrival will also be insignificant (it becomes negligible). After controlling for volatility persistence, Hiemstra and Jones (1994) find evidence for the existence of nonlinear causality from volume to returns.

Gervais *et al* (2001) find that extreme trading activities contain information about the future evolution of stock prices. When innovations to the information arrival process and information innovations in the equilibrium price change process are equal, theoretically, the speed of evolution in both the price change process and the information arrival process should be equal (empirically approximately equal), in order for the mixing variable to drive evolutions in return and volume. Any revision in the direction of the covariance between equilibrium market price (index) changes and the equilibrium stock price changes provides the best guidance as to whether the trading volume is a measure of speed of evolution in the price change process that could be picked up by the ARCH models used for capital asset return modelling.

Bauer and Nieuwland (1995 p.140) quote that 'in Lamoureux and Lastrapes (1990) innovation to the mean equation (equation 1, p.222) were conditionally normal, where the trading volume as a proxy for information arrival was contained in the information set.' If this is valid, then ε_t in equation (1) should be replaced by u_t in equation (11) as innovations in the price change process to motivate the theoretical arguments of this study.

By substituting *white noise* u_t in the equation (11) into the *error term* ε_t in equation (10), the representation would yield:

$$\text{Covariance } (R_{it}, R_{mt}) = \left(\frac{r_{it} - u_t}{r_{mt}} \right) \sigma_{mt}^2 \quad (12)$$

2.4 Revision of expectation

When there is no new information arrival at the market, trading is slow and the innovation process evolves slowly. When new information arrives at the market, active trading with innovations in the price change process can be observed. Accordingly, if n_t is the directing process, the distribution of increments (innovations) in the price change process would have a distribution subordinate to that of the price changes and directed by the distribution of trading volume (See e.g. Clark 1973 p.142). Stock volume is a noisier or fuzzier indicator of the flow of information arrival at the market than price volatility (See e.g. Shalen 1993). Each and every time new information violates old expectations, innovations to the equilibrium price increments should increase and the speed of evolution in the price change process and the directing process then depends on whether the trading is slow or fast.

The covariance is higher when the market return is strongly correlated with the stock return. Assume a stock whose return and the market return are strongly correlated and which is trading in an efficient market (See Fama 1965, 1970). When the information flows into the market at such a rate (proxied by volume) investors respond to such 'new' information by adjusting their investment strategy. Expectations of investors are deviated from the general (common) expectation of the market (i.e. expectation of market return in the absence of new information about the stock being traded), as a symmetric response to the information flows into the market (See especially Clark 1973 pp.144-145; Epps and Epps 1976 p.307 and p.309, for similar arguments). It is clearly documented that price volatility may be positively related to the dispersion of expectation (See e.g. Pflleiderer 1984; Shalen 1993). The expectations of investors after the rate of information flow is taken into account (as captured by ARCH), deviate from the market expectation, hence a change in the covariance between market return and the stock return is expected. In the absence of new information, investors have a homogeneous or common expectation.¹² Arguably, if there is no new information that affects stock prices, the investors do not want to adjust or change their current investment strategy (buy and sell) but to accept what the market offers. It is unarguably true that the market return is the opportunity cost of foregoing any stock market investment. Suppose an investor, without possessing any sort of information about the stocks, arrives new to invest in the stock market. Such an investor expects at least the market return, even before the valuation and selection of stocks. Further investigation of these behavioural matters is left for future researchers. On the other hand, in an efficient market, market prices already incorporate and reflect all relevant/available information at any given point of time (See e.g. Fama 1970). Thus trading without 'new' information does not make sense. Those who believe in the efficient market hypothesis take the view

that it is pointless to search for undervalued stocks, or try to predict trends in the market, through fundamental analysis or technical analysis. The question one would then have to ask is why people trade in the stock market. The answer to this question is convincingly documented in this study.

Including innovations in the information arrival process into the equation 12, the covariance between market return and the stock return could also be represented as:

$$\text{Covariance } (R_{it}, R_{mt}) = \left(\frac{r_{it} - (n_t - b(L)n_{t-1})}{r_{mt}} \right) \sigma_{mt}^2 \quad (13)$$

Tauchen and Pitts (1983) and many others identify volume and price changes as a joint function of information flows into the market. As such, return innovation or surprise alone, as noted in Engle (1982), Engle *et al* (1987) is sufficient to conclude on the time dependence in the rate of information arrival at the market in capital asset pricing. Nonetheless, such an argument might depend upon operational time.

3. DATA AND METHODOLOGY

3.1 *Sample and sampling procedure*

The statistical population of this study consists of all firms listed in the Colombo Stock Exchange (CSE) that were traded during the time period 2010 to 2013 (year ending 31st December). Return¹³ and volume data have been obtained from the CSE publications. Out of 292 listed companies (as at 31st October 2014) in the CSE, 20 companies are selected on a random sampling basis subject to the following criteria:

1. The shares of companies should have been actively trading during the period 4th January 2010 to 31st December 2013 on the Colombo Stock Exchange. High-ranked actively trading stocks (companies) are given priority in the sample selection. The selection of actively trading stocks mitigates the possible effect of any negative correlation between lagged returns and contemporaneous volatility.
2. The shares of companies paying higher dividends¹⁴ (i.e. the stocks with high dividend yield) are eliminated from the sample.
3. The shares subject to share splits and reverse splits (consolidation of shares) during the sampling period are dropped from the sample to eliminate the potential effect on share price.

3.2 *Contemporaneous stock volume*

There is a possibility of simultaneity bias, as the volatility h_t and contemporaneous volume occur in the same time period t .¹⁵ However a large literature has demonstrated that there is a positive contemporaneous correlation

between stock volume and return volatility (see e.g. Karpoff 1986; Lamoureux and Lastrapes 1990¹⁶; Shalen 1993; Andersen 1996) and the stock volume is related to the return volatility estimation process. Clark (1973) imposes restrictions on the contemporaneous return-volume relationship in the mixture of the distribution structure. However, Mandelbrot (1963) documents that squared returns are positively and serially correlated, demonstrating such as a salient feature of stock return data. Scholars such as Clark (1973), Tauchen and Pitts (1983), Foster and Viswanathan (1990), Gallant *et al* (1992), and Lamoureux and Lastrapes (1994), have demonstrated that there is a strong correlation between squared returns and stock volume.

3.3 The estimation of the model — Generalised Autoregressive Conditional Heteroskedasticity (GARCH) for stock returns

The GARCH estimation model for capital asset pricing is given below. The contemporaneous volume V_t is introduced into the conditional variance in equation (15).

$$r_{it} = \beta_0 + \beta_m r_{mt} + \varepsilon_t \quad \text{recall (1)}$$

$$\varepsilon_t \setminus (V_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \dots) \sim N(0, h_t), \quad (14)$$

$$h_t = \alpha + \pi(L)\varepsilon_{t-1}^2 + \lambda(L)h_{t-1} + \mathbb{I}V_t \quad (15)$$

Where \mathbb{I} is the volume coefficient and V_t is the volume of trades of individual stocks. It is expected that $\mathbb{I} > 0$. Also, π and λ (the total variance persistence as captured by $(\pi + \lambda)$) are expected to be insignificant when accounting for the uneven flow of information under serial correlation.

4 EMPIRICAL FINDINGS AND DISCUSSION

4.1 Preliminary analysis of sample data

Table I reports the empirical properties of stock returns and volume. The JB test statistic is higher for all companies in the sample, demonstrating non-normality of the unconditional distribution of daily returns. Nonnormality of return distribution is also witnessed by the observation of kurtosis and skewness in the returns distribution. Kurtosis exceeds 3 in all 20 companies and skewness exists. The null hypothesis of no ARCH effect in the data, as in the ARCH-LM test, is rejected. ARCH effect in returns exists for 19 companies in the sample, as the test statistic exceeds the critical value of 7.815 at the 5 per cent significance level. The results of the Box-Ljung Q statistic are statistically significant for 9 companies in the sample, displaying serial correlation in volume series. The test statistic of these companies exceeds the critical value of 31.41 in χ^2 (20) distribution at the 5 per cent significance level.

Table I: Empirical Properties of Daily Stock Returns and Volume

Company	T	Period	Skewness	Kurtosis	^a JB	^b LM	^c Q(20)
Acme Printing & Packaging	948	1/4/10-12/31/13	0.23	6.46	480.51	65.697	10.806
Browns Investments*	589	7/26/11-12/31/13	0.95	6.41	373.86	16.659	1.370
Seylan Developments	959	1/4/10-12/31/13	1.37	9.48	1978.99	14.969	221.050
CT Land Development	919	1/4/10-12/31/13	0.60	4.55	148.12	52.098	30.612
East West Properties	958	1/4/10-12/31/13	2.12	19.37	11408.74	15.211	85.870
Panasian Power*	426	1/7/11-12/31/13	9.14	168.10	828508.80	0.101	101.010
Kingsbury	945	1/4/10-12/31/13	0.46	9.33	1611.52	66.834	4.473
F L C Holdings*	655	4/18/11-12/31/13	0.76	6.51	400.78	25.099	14.971
Sierra Cables	959	1/4/10-12/31/13	0.97	7.42	931.37	11.320	51.639
Tess Agro	959	1/4/10-12/31/13	1.14	8.03	1219.84	12.589	129.470
Textured Jersey Lanka*	580	8/9/11-12/31/13	0.70	6.93	420.73	22.087	14.830
Expolanka Holdings*	619	6/13/11-12/31/13	0.57	6.87	420.90	16.635	51.381
People's Leasing & Finance*	509	11/24/11-12/31/13	0.79	11.71	1662.71	8.308	34.225
John Keells Holdings	483	1/2/12-12/31/13	(0.82)	10.90	1311.92	16.314	1.787
PC Pharma*	421	1/18/12-12/31/13	0.39	7.96	442.15	17.689	3.710
Taprobane Holdings*	302	5/17/12-12/31/13	0.14	5.86	103.77	19.593	25.412
Commercial Bank of Ceylon	483	1/2/12-12/31/13	0.31	5.76	160.44	16.800	48.486
National Development Bank	483	1/2/12-12/31/13	0.59	9.84	969.29	28.322	4.744
Cey. Hotels Corporation	940	1/4/10-12/31/13	0.83	7.10	770.40	28.930	18.886
CIC Holdings (Voting)	912	1/4/10-12/31/13	1.10	12.63	3707.04	8.840	82.668

1. The number of observations (T) is reported against respective periods in which data occurred for each company. JB is the Jarque-Bera test statistic for normality. LM is the ARCH LM test statistic for the number of observations multiplied by the R-squared value for 3 lags. Q (20) is the Ljung-Box Q statistic for serial correlation upto 20 lags, in the volume series.

2 ^a Under the null hypothesis for normality, the critical value of $\chi^2(2)$ distribution at 5% significance level is 5.99

3. ^b Under the null hypothesis, the critical value of $\chi^2(3)$ distribution at 5% significance level is 7.815

4. ^c Under the null hypothesis for no serial correlation, the critical value of $\chi^2(20)$ distribution at 5% significance level is 31.41

5.*Recently listed companies

4.2 ARCH and GARCH effect

Table II summarises the estimation output of the maximum likelihood GARCH (1, 1) model *without* stock volume included in the conditional variance equation (equations (3) of the conceptual model). Except for one company (Panasian Power, in which the ARCH term becomes insignificant at 5 per cent

significance level), the coefficients β_m , π , and λ are statistically significant at 5 per cent significance level for all companies. Table III shows the parameter estimation with *stock volume* being included in the conditional variance equation of GARCH (1, 1) model (equation (15) of the estimation model).

After stock volume is included in the conditional variance equation of the GARCH model, the ARCH term becomes insignificant for 15 companies in the sample (See Table III) at the 5 per cent significance level (equation (15)). The finding is consistent with the prior findings of Lamoureux and Lastrapes, (1990). This also provides support for the hypothesis that ARCH is a reflection of time dependence in information arrival; thus the frequency of observation matters in ARCH modeling (See especially Lamoureux and Lastrapes 1990). These results also imply that the speed of evolution in the information arrival process of such companies is expected to be active, as may be evidenced by the 'volume clock'¹⁷ and may be a good measure of the speed of evolution in the price change process, so that an estimator may use the ARCH model (as in GARCH (1, 1)) with confidence in the precision of the estimate of returns. However, the way to test this hypothesis under heteroscedasticity is to examine the nature of the response of market participants to the evolution in the information arrival process. Empirical findings in support of this hypothesis are discussed in Section 4.3. As Table III reports, the volume coefficient is positive for 12 companies in the sample. These results provide support for the hypothesis that the conditional variance of daily price changes is positively related to the volume. A negative correlation between volatility and volume may be associated with sudden jumps (non-continuous) in stock prices, for a variety of reasons (see, for example, Amatyakul 2010; Giot *et al* 2010 and Wang and Huang 2012 pp.212-213 for useful discussions).

Investors receive information at varying rates on different days. When large volumes are supported by active trading, the speed of evolution in the information arrival process, as directed by trading volume as seen in equation (5), is strongly correlated with the speed of evolution in the price change process. For companies whose information arrival process evolves slowly, the price change process reacts accordingly but with deficiencies.

The randomly selected sample includes eight recently listed companies. Information evolution is expected to be higher in recently listed companies than matured companies, where the investors are in receipt of information progressively through the exchange announcements and by the media release in the first few years of operation. Thus, high-frequency observations in the volume and return can be observed in such companies. These companies can be perfect candidates for ARCH modeling as it might more precisely capture time dependence in the process of information arrival. However these results, along with the findings above, should be interpreted in conjunction with the linkage between volatility persistence, which captures the evolution in the information arrival process, and the price change process.

Table 2: Maximum Likelihood Estimation of GARCH (1,1) Model without Volume

<i>Company</i>	β_m	<i>z-Stat</i>	<i>p value</i>	π	<i>z-Stat</i>	<i>p value</i>	λ	<i>z-Stat</i>	<i>p value</i>
Acme Printing & Packaging	0.079	14.993	0.000	0.147	7.015	0.000	0.831	39.670	0.000
Browns Investments	1.705	15.179	0.000	0.152	5.976	0.000	0.770	26.545	0.000
Seylan Developments	1.895	26.800	0.000	0.098	7.351	0.000	0.882	50.822	0.000
CT Land Development	1.303	16.325	0.000	0.113	4.343	0.000	0.733	10.702	0.000
East West Properties	1.905	31.728	0.000	0.248	16.155	0.000	0.734	52.806	0.000
Panasian Power	1.249	6.068	0.000	0.004*	0.529	0.597	0.839	4.173	0.000
Kingsbury	0.692	9.639	0.000	0.132	8.318	0.000	0.822	48.809	0.000
F L C Holdings	0.093	12.661	0.000	0.158	5.143	0.000	0.757	16.525	0.000
Sierra Cables	1.385	12.620	0.000	0.217	6.606	0.000	0.515	8.608	0.000
Tess Agro	1.676	13.034	0.000	0.267	9.747	0.000	0.402	7.508	0.000
Textured Jersey Lanka	1.244	18.743	0.000	0.060	3.774	0.000	0.887	27.658	0.000
Expolanka Holdings	1.143	16.609	0.000	0.049	3.809	0.000	0.887	95.690	0.000
People's Leasing & Finance	0.741	9.518	0.000	0.149	9.615	0.000	0.766	30.850	0.000
John Keells Holdings	0.766	22.435	0.000	0.435	6.130	0.000	0.399	5.598	0.000
PC Pharma	1.400	4.561	0.000	0.228	6.616	0.000	0.751	27.091	0.000
Taprobane Holdings	1.166	3.979	0.000	0.164	4.336	0.000	0.768	22.010	0.000
Commercial Bank of Ceylon	0.437	13.902	0.000	0.239	4.189	0.000	0.562	7.052	0.000
National Development Bank	0.831	13.795	0.000	0.146	3.333	0.001	0.526	4.333	0.000
Cey. Hotels Corporation	1.418	14.898	0.000	0.123	4.893	0.000	0.623	7.797	0.000
CIC Holdings	1.065	16.167	0.000	0.163	5.781	0.000	0.684	17.027	0.000

Notes:

1. Mean equation $r_{it} = \beta_0 + \beta_m r_{mt} + \varepsilon_t$ and conditional variance equation $h_t = \alpha + \pi(L)\varepsilon_{t-1}^2 + \lambda(L)h_{t-1}$

2. The coefficients for all companies are statistically significant at 5%, assuming returns are conditionally normally distributed except for Panasian Power *

4.3 Symmetric response of investors

After the stock volume is included in the conditional variance equation (15) of the GARCH (1, 1) model, the average covariance between return on market portfolio and stock return is increased in 15 companies;¹⁸ in the sample in which 13 companies, ARCH term becomes insignificant at 5 per cent level of GARCH (1, 1) model (See Table IV). This provides strong evidence for the hypothesis that volume is a measure of speed of evolution in the price change process, while giving no conclusion on the speed of revision of the expectations of investors. For these companies, the information arrival process (information innovations) evolves faster and they are good candidates for return modeling using ARCH models.¹⁹ Any symmetric response from market participants provides evidence as to whether the ARCH captures this mixed distribution property in return modeling.

Table 3: Maximum Likelihood Estimation of GARCH (1,1) Model with Volume

<i>Company</i>	β_m	<i>z-Stat</i>	<i>p value</i>	π	<i>z-Stat</i>	<i>p value</i>	λ	<i>z-Stat</i>	<i>p value</i>	Π	<i>z-Stat</i>	<i>p value</i>
Acme Printing & Packaging	0.073*	7.366	0.000	0.150	1.570	0.117	0.600*	2.495	0.013	-1.39E-11*	-3.224	0.001
Browns Investments***	1.750*	13.762	0.000	0.120*	2.415	0.016	0.014	0.475	0.635	3.23E-10*	5.399	0.000
Seylan Developments***	1.620*	6.288	0.000	0.150	1.289	0.197	0.600*	2.350	0.019	0.00E+00	0.000	1.000
CT Land Development	1.347*	16.127	0.000	0.062	1.573	0.116	0.079	0.468	0.640	3.22E-09*	3.376	0.001
East West Properties	1.334*	17.121	0.000	0.045	1.503	0.133	-0.026	-1.203	0.229	3.39E-09*	15.168	0.000
Panasian Power***	1.270*	2.215	0.027	0.150	0.861	0.389	0.600*	2.066	0.039	0.00E+00	0.000	1.000
Kingsbury***	0.904*	4.010	0.000	0.150**	1.679	0.093	0.600*	2.296	0.022	0.00E+00	0.000	1.000
F L C Holdings	0.121*	5.728	0.000	0.150	1.246	0.213	0.600**	1.930	0.054	-7.78E-13*	-31.539	0.000
Sierra Cables***	1.409*	4.255	0.000	0.150	1.181	0.238	0.600*	2.084	0.037	0.00E+00	0.000	1.000
Tess Agro***	1.626*	3.975	0.000	0.150	1.130	0.259	0.600*	2.018	0.044	0.00E+00	0.000	1.000
Textured Jersey Lanka	1.216*	20.479	0.000	0.070*	2.289	0.022	-0.026*	-2.104	0.035	3.85E-10*	7.824	0.000
Expolanka Holdings***	1.229*	6.893	0.000	0.150	1.292	0.196	0.600*	5.664	0.000	-3.20E-11*	-119.626	0.000
People's Leasing & Finance	0.743*	10.072	0.000	0.105*	2.555	0.011	-0.031	-0.715	0.475	7.03E-10*	7.348	0.000
John Keells Holdings	0.963*	7.636	0.000	0.150**	1.761	0.078	0.600**	1.807	0.071	-3.68E-12*	-3.931	0.000

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PC Pharma	1.737**	1.748	0.080	0.150**	1.751	0.080	0.599*	3.755	0.000	-2.37E-10*	-18.318	0.000
Taprobane Holdings	1.453	1.515	0.130	0.150	0.822	0.411	0.600	1.307	0.191	-1.45E-09*	-2.085	0.037
Commercial Bank of Ceylon	0.504*	6.877	0.000	0.150	1.160	0.246	0.600*	1.978	0.048	-4.79E-12*	-8.355	0.000
National Development Bank	0.859*	5.165	0.000	0.150	1.225	0.221	0.600*	3.520	0.000	-7.22E-11*	-33.097	0.000
Cey. Hotels Corporation	1.336*	13.900	0.000	0.247*	7.256	0.000	-0.004	-0.398	0.691	6.14E-09*	6.064	0.000
CIC Holdings	0.960*	15.579	0.000	0.257*	6.890	0.000	0.199*	4.136	0.000	1.29E-09*	10.060	0.000

Notes:

1. Mean equation $r_{it} = \beta_0 + \beta_m r_{mt} + \varepsilon_{it}$ and the conditional variance equation $h_t = \alpha + \pi(L)\varepsilon_{t-1}^2 + \lambda(L)h_{t-1}$.
 2. *Statistically significant at 5% assuming returns are conditionally normally distributed. **Statistically significant at 10%
 3. ***Significantly high volumes of daily trades were observed during the sampling period.
 4. Convergence tolerance is set at (default: 1E-04) thirteen zero-digits by Eviews.
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When the 'new' information arrives on the market, investors learn about the information relating to stock price movements and choose to trade around the fair value. Ultimately investors settle around the right (equilibrium) price after learning²⁰ themselves of relevant information flows into the market, hence decreased information risk. As information flows into the market, traders/investors will tend to find for themselves, or learn from market sources, what might have an impact on stock prices. This reduces the risk of trading in securities without relevant information. Information arrival at the market is a stochastic process with positive increments, in the sense of Clark (1973), which results in new information being available to investors.²¹ Thus, when information arrival rate is accelerated or arrival time is frequent, novel information will be available to investors in this positive increment process.

When there is no new information available to investors, as evidenced by slow trading, the expectations of investors are moving towards market expectations. The price change process always occurs, irrespective of whether trading is active or slow, but with a deficiency in the volume commensurate with the price change. Thus, this relationship is worth accounting for in asset pricing incorporating heteroskedasticity. When trading is active and arrival of information is speedier, the expectations of investors are revised and the covariance (co-movements) between market price changes (index return) and the stock return is decreased; thus it deviates from the market expectation or the common expectation of all investors. When new information pertaining to a particular stock flows into the market, the investors who hold stocks or who intend to buy/sell stocks adjust their trading/investment strategy accordingly, in response to the information available in the market. This is well justified by the results of the equation in endnote (18) as reported in table IV.

Under the efficient market hypothesis, the buying and selling of securities does not make sense unless one is playing for a stroke of luck. As such, investors or participants in the market trade in stocks with expectations based on new information flows into the market.²² This argument is in line with the thesis of Bachelier (1900)²³ in which he argues that alert speculators will receive no information from past prices. Also, Fama *et al* (1969) find evidence on the speed of adjustment of market prices to new information and conclude that the stock market is efficient as stock prices adjust very rapidly to new information.²⁴ If there is no new information flow present in the market, investors tend to follow the market on average. Therefore, there are two sets of investors emerging in an efficient stock market:

1. Uninformed²⁵ Market Followers.

The investors who accept and seek market return or who are satisfied with what the market offers in the absence of new information flows into the market. When information flow is not available²⁶ or the information arrival process evolves slowly, expectations of investors move towards the market expectation (market return) on average. Market return in some sense is an

indication of the common expectation of all investors, which closely resembles the aggregate amount of information arriving at the market, and can be proxied by the stock volume of individual companies. Rational investors will switch their investment funds based upon what offers the best return. For example, a bank depositor may see investment in stock market as an opportunity that gives him a greater return than what banks offer. The depositor may then withdraw money from the bank and invest in a given stock following appropriate valuation and due appraisal. However, what may still be in the investor's mind is what the market offers to such an investor, unless the investor comes up with some special (i.e. new) information relevant to the securities in which the investment is sought. Thus, all actions (buying and selling) performed by investors pertaining to a particular stock are reflected in equilibrium stock price changes, which co-vary closely with market price (index) changes.²⁷ These investors have a common expectation, instead of expectations conditional upon firm-specific information arrival at the market. Investors may even trade in the absence of new information arrival at the market but for a common expectation²⁸ (i.e. the market expectation). Uninformed market-followers trade on information variables that are largely associated with systematic risk which cannot be diversified away and which is beyond the control of individual companies (See endnote 12).

2. Informed Market Deviants.

When new information arrives at the market, investors adjust their investment strategy (buy or sell) in response to the information flows into the market, so that each such action leads them to deviate their expectation from the market expectation towards what arises out of the information flows into the market. Factors relating to firms' specific information flows are associated with unsystematic risk and are within the control of individual firms. Informed market deviants trade on firm-specific new information arrival at the market. However, firm-specific information flows may be superseded by the information variable relating systematic risk in an unsettled market with investor panic (e.g. a financial crisis). When the new information arriving at the market accelerates, the extent to which the equilibrium price changes of individual stocks co-vary with that of the equilibrium market return tends to be decreased.

5. CONCLUSIONS

This article revives interest in the ARCH modeling strategy for capital asset pricing under mixture of distribution framework. The hypothesis, that ARCH is a manifestation of daily time dependence in the rate of information arrival at the market in capital asset pricing, is tested by observing changes in the behaviour of investors in response to new information flows into the market. This provides evidence for the argument under ARCH modeling that expected stock returns (especially in CAPM) should be generated by a mixture of distribution. Thus, ARCH errors should also be drawn from a mixture of distribution

Table 4: Decomposition of Covariance (on average)

Company	Variance GARCH (without Volume)	Variance GARCH (with Volume)	σ_t (without Volume)	σ_t (with Volume)	σ^2_{mt}	σ_{mt}	Correlation between r_t and r_m (Constant)	Covariance between r_t and r_{mt} (without Volume)	Covariance between r_t and r_{mt} (with Volume)
Acme Printing & Packaging	7.79E-05	2.10E-04	8.23E-03	1.44E-02	1.49E-03	3.52E-02	0.309145	9.44E-05	1.57E-04
Browns Investments	7.82E-04	1.10E-03	2.70E-02	2.74E-02	6.77E-05	7.68E-03	0.509946	1.06E-04	1.07E-04
Seylan Developments	7.78E-04	2.06E-03	2.62E-02	4.52E-02	8.28E-05	8.47E-03	0.481881	1.12E-04	1.86E-04
CT Land Development	7.74E-04	7.72E-04	2.74E-02	2.69E-02	8.63E-05	8.68E-03	0.411019	9.96E-05	9.64E-05
East West Properties	1.63E-03	1.63E-03	3.49E-02	3.08E-02	8.28E-05	8.48E-03	0.350021	1.11E-04	9.54E-05
Panasian Power	1.75E-03	5.03E-03	4.18E-02	6.95E-02	7.21E-05	7.98E-03	0.251613	8.39E-05	1.40E-04
Kingsbury	7.01E-04	1.88E-03	2.50E-02	4.31E-02	8.39E-05	8.54E-03	0.308080	6.66E-05	1.14E-04
F L C Holdings	5.45E-05	1.48E-04	7.07E-03	1.21E-02	6.61E-05	7.67E-03	0.474395	2.83E-05	4.51E-05
Sierra Cables	1.09E-03	2.97E-03	3.20E-02	5.43E-02	8.28E-05	8.47E-03	0.370146	1.00E-04	1.70E-04
Tess Agro	1.61E-03	4.48E-03	3.87E-02	6.66E-02	8.27E-05	8.47E-03	0.350482	1.14E-04	1.98E-04
Textured Jersey Lanka	2.46E-04	2.63E-04	1.55E-02	1.49E-02	6.53E-05	7.56E-03	0.564059	6.74E-05	6.45E-05

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Expolanka Holdings	2.66E-04	6.83E-04	1.58E-02	2.60E-02	6.76E-05	7.71E-03	0.536819	6.67E-05	1.08E-04
People's Leasing & Finance	3.02E-04	3.02E-04	1.67E-02	1.57E-02	6.13E-05	7.29E-03	0.393354	4.86E-05	4.61E-05
John Keells Holdings	1.44E-04	3.55E-04	1.11E-02	1.88E-02	5.80E-05	7.15E-03	0.560149	4.62E-05	7.60E-05
PC Pharma	5.17E-03	1.09E-02	6.60E-02	1.04E-01	6.75E-05	7.71E-03	0.211520	1.17E-04	1.72E-04
Taprobane Holdings	2.02E-03	5.40E-03	4.31E-02	7.32E-02	6.21E-05	7.56E-03	0.257102	8.58E-05	1.43E-04
Commercial Bank of Ceylon	6.12E-05	1.58E-04	7.58E-03	1.25E-02	5.80E-05	7.15E-03	0.463948	2.55E-05	4.17E-05
National Development Bank	1.98E-04	5.59E-04	1.38E-02	2.35E-02	5.80E-05	7.15E-03	0.431433	4.29E-05	7.28E-05
Cey. Hotels Corporation	9.10E-04	1.37E-03	2.97E-02	3.02E-02	8.50E-05	8.59E-03	0.419151	1.08E-04	1.11E-04
CIC Holdings (Voting)	5.26E-04	5.06E-04	2.21E-02	2.14E-02	8.69E-05	8.74E-03	0.390208	7.69E-05	7.48E-05

Note:

The estimation equation $Cov(r_t, r_{mt}) = Correlation(r_t, r_m) \sigma_t \sigma_{mt}$ where σ_t is the standard deviation of return of stock at time t and σ_{mt} is the standard deviation of return on the market portfolio at time t .

in order to use ARCH models efficiently in capital asset pricing. These findings strongly encourage the use of ARCH models for return modeling in asset markets.

The theory of random walk stems from the argument that asset prices only adjust to new information, as existing information is already reflected in market prices. When new information arrives at the market, a revision in the direction of expectation which deviates from the market expectation is observed. In the absence of new information arrival at the market, investors follow the market on average. Accordingly, two sets of investors, namely informed market deviants and uninformed market followers, are hypothesised to exist in an efficient market whose behaviour is dependent upon the arrival of new information at the market. This provides a theoretical base for the hypothesis that successive price changes are independent and generated from a mixture of distribution based upon the arrival of new information at the market. This article opens up many avenues for future research, for example the payoffs accruing to equity holders from the timing of corporate announcements and cash flows may differ substantially from a followers' market to a deviants' market.

As examined by Lamoureux and Lastrapes (1990) the form of heteroscedasticity (ARCH) in stock returns is based upon the choice of observation frequency. This hypothesis is testable when stock volume measures the speed of evolution in price change process. This is dependent upon how fast the volume clock evolves, given the speed of evolution in the information arrival process. This article demonstrates the ability of ARCH models to account for the mixed distribution properties of stock returns in capital asset pricing (i.e. modelling return). Nonetheless, the form of heteroscedasticity accounted for by ARCH is a matter of operational time which, in turn, will determine the precision (i.e. closeness to the true value) of return estimation.²⁹

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ENDNOTES

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2. The fundamental assumption about information flow in this study is that the information flow, as proxied by stock volume, is always relevant to stock price changes and the involvement of rational investors in the trading process in an efficient market.

3. Ying (1966 p.676) points out that 'Prices and volumes of sales in the stock market are joint products of a single market mechanism, and any model that attempts to isolate prices from volumes or vice versa will inevitably yield incomplete if not erroneous results'. This motivates testing for heteroskedastic mixture in capital asset pricing. See also Andersen (1996 p.187) for a discussion of the advantages of utilising trading vol-

ume figures in conjunction with returns in modeling volatility.

4. Speed of evolution refers to the speed of information evolution.

5. The hypothesis that tests whether the ARCH accounts for this mixing property of a given sample of stock returns on which the ARCH model is applied for return modeling (i.e. under capital asset pricing framework when an exogenous variable is introduced in the mean that predicts return).

6. Clark (1973 pp.144-145) argues 'when new information (in the form of data that the traders consider relevant) flows to the market, both prices and traders' price expectations will change'. He also points out that 'all traders would revise their expectations in the same direction'.

7. Here, r_{mt} is an exogenous variable introduced along with its vector β_m . However, Lamoureux and Lastrapes (1990) constrain the mean conditional upon past information to zero.

8. The validity of this assumption in capital asset pricing is also tested (See endnote 5). Equation 4 under this framework is constructed on the assumption of perfect mixture (i.e. complete subordination between ε_t and δ_t). See also Equation 12.

9. Andersen (1996) points out that the joint distribution of price changes and informed trading volume is identical over each period and is therefore identically and independently distributed (i.e. i.i.d). Furthermore, Epps and Epps (1976) assume that ε_t is identically normally distributed with zero mean and constant variance. It is assumed that the *white noise/error term* of equations (1), (5), and (11) of the conceptual model are identically and independently distributed, assuming the duty in their mean representations in which the law of large numbers and central limit theorem apply. See section 2.3 for an extensive discussion.

10. The standard assumptions of the Efficient Market Hypothesis and CAPM do apply. Information innovations in the information arrival process may also include innovations on information variables relating to common expectation, as pointed out in endnote 12. However, such effects are standardised in the mean, as in Equation 12 or 13. As such the decomposition of information flow is unnecessary.

11. See e.g. Fama and French (1992), Fama and French (1996) for the meaning incidental to the factor loading.

12 A new information process evolves when there are innovations in the information arrival process as $n_t > b(L)n_{t-1}$. When $n_t = b(L)n_{t-1}$, there is no novelty in the information arrival process. Thus, trading when there is no new information arrival at the market (when there is no information innovation in information arrival process) will be for a common expectation. Common expectation is largely associated with the information variables relating to systematic risk that are beyond the control of individual firms, for example economic and political factors. These factors affect all firms in general.

13. All Share Price Index data were not available for 8th of March 2010. Therefore the return and the volume data of each stock have been eliminated from the computations.

14. The stock volume and return volatility relationship may be affected by the dividend, induced trading around ex-dividend dates, as the literature demonstrates in other settings. For taxable distributions, trading volume tends to be increased significantly around the ex-dividend dates. This principle is more applicable for high dividend paying companies. As such the arguments under the conceptual framework may be affected by the effect of tax induced trading around ex-dividend dates, as points out by Lakonishok, and Vermaelan (1986).

15. Fleming *et al* (2005 p.2) argue that 'MDH implies that the impact of simultaneity bias becomes negligible as the number of traders in the market and/or the number of daily information events becomes large'.

16. Lamoureux and Lastrapes, (1990) emphasise that lagged volume had poor explanatory power in the conditional variance equation. It is also noted that any study that regresses return volatility on volume is subject to simultaneity bias, if stock volume is not exogenous (See. e.g. Karpoff 1986). Stock volume is weakly exogenous in the sense of Engle *et al* (1983).
17. Volume clock, for the purpose of this study, means the clock (proxied by volume) assigned to measure the speed of evolution in the information arrival process. To what extent this clock is related to the speed of evolution in the price change process, that ARCH used for capital asset pricing should account for, is tested.
18. Equation used for the decomposition of covariance is: $Cov(R_{it}, R_{mt}) = Correlation(R_i, R_m) \sigma_{mt} \sigma_i$
19. Inclusion of stock volume in the conditional variance equation means, neutralising (or making negligible) the effect of time dependence (as total volatility persistence captures the time dependence) in the rate of information arrival, which is reflected in the conditional volatility.
20. Andersen (1996 p.172) notes that 'private information arrivals induce a dynamic learning process that results in prices fully revealing the content of the private information through the sequence of trades and transaction prices'.
21. Provided information innovation exists when $n_t > b(L)n_{t-1}$. Note that $r_{it} \geq u_t \geq 0$ conditional upon n_t and $r_{it} = u_t$ under a perfect mixture. See also Equation 12 and Endnote 12.
22. New information flow indicates that the flow is always a product of new information, as each flow of information violates old expectations and revised or fresh expectations are formed with the arrival of this information.
23. Referenced the translated version, Bachelier, L. (2011).
24. The use of contemporaneous stock volume as a proxy for the rate of information arrival at the market is well justified within the framework of Fama (1965) and Fama *et al* (1969).
25. Investors are informed when there are innovations in information arrival process. See Endnotes 12 and 22.
26. See Endnotes 10, 12 and 22 for precise explanations of information arrival and new information arrival.
27. See also the limitations, discussed in Section 3.2
28. Large clusters of index changes are therefore apparent in equity markets. See also the assumptions made under Endnote 2.
29. This is well addressed by Andersen and Bollerslev (1998) in relation to forecasting volatility.

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