Long-phased Marx-Goodwin profit- and wage-squeeze cycles in wage-led economies

Matthieu Charpe, Peter Flaschel and Christian R Proaño

ABSTRACT

A widely debated issue in heterodox economics is the question of whether macroeconomic activity reacts positively or negatively to increases in the wage share, i.e. whether it is wage- or profit-led. In the present paper, from an empirical perspective, we show that this question is of secondary importance for Marx’s model of the distributive cycle. Our analysis starts with the traditional Goodwin (1967) model – which describes the dynamic interaction between the wage share and the employment rate – to which we add an effective demand function to cover the utilisation of the capital stock (thus a Keynes-component to the original supply-side dynamics). In this extended Goodwin model we show that the Goodwin story remains, qualitatively, akin to Marx’s supply-side model, although the distributive cycle will now also depend on the state of effective demand. Here we find that the question of whether the capacity utilisation of firms is driven by a profit-led or a wage-led goods-market regime is irrelevant if a mild elasticity condition in the case of a positive dependence of the capacity utilisation rate on the wage share is met. We illustrate this result from an empirical perspective.

1. Introduction: Goodwin’s MKS System

In a symposium at Groningen University (later published in a corresponding conference volume) Goodwin described what he considered his fundamental contribution to macroeconomics, namely the conceptualisation of the dynamics of capitalism through the combination of elements of the thinking of three key economic scholars: Marx, Schumpeter and Keynes (Goodwin 1989 p.52). Marx is included for his characterisation of the dynamics of profits and accumulation in capitalist economies, Schumpeter for his analysis of the role of innovation as the driving force of capitalism, and Keynes for his analysis of the role of effective demand in the determination of the level, and rate of change, of output.

Although seemingly incompatible bedfellows (in logical time), Goodwin nevertheless viewed their theories as being complementary to each other. In the Marxian distributive cycle, the wage share \((v)\) and the rate of employment \((e)\)
interact with each other in an overshooting way, as first described in rigorous terms by Goodwin (1967). In Keynes’s (1936) ‘General Theory’ (when viewed from the perspective of a growing economy) the actual output-capital ratio \((y, \text{ the rate of capacity utilisation})\) interacts dynamically with (an array of) interest rates \((i)\) determined by financial markets. In the context of the distributive cycle, this interaction generally causes downward deviations from the actual \((y)\) to the potential output-capital ratio \((y^p)\). Furthermore, the evolution of the potential ratio is the subject of Schumpeter’s theory of technical change, alongside the evolution of labour productivity \((z)\). According to Schumpeter, the former is subject to long waves, instead of a secularly falling \(y\), as Marx seems to have suggested. The latter may be explained since Marx only had the first complete long wave, industrialisation, at his disposal as a historical example.²

In our view Goodwin’s MKS (Marx-Keynes-Schumpeter) system represents the most robust basis for future work on the laws of motion of capitalism. The present paper therefore attempts to go in such a direction, in particular developing a Keynesian version of the Marxian distributive cycle. In this paper we consider the role of wage-led demand – which will be shown to be of secondary importance in the shaping of the Marxian distributive cycle – and focus on the variables which characterise the Goodwin model, \(v\) and \(e\). We shall add an effective demand function for the utilisation of the capital stock. Thus, the Keynesian variable \(y\) is introduced for consideration. Financial markets and Schumpeterian technical change will be kept constant, in the background of the model, in the form of a given rate of interest, a given potential output-capital ratio, and a given state of labour productivity. In this simple Keynesian extension of the Goodwin model we show that the Goodwin story remains qualitatively the same as in his supply-side approach, although the distributive cycle will now also depend on the state of effective demand at each moment in time, with only a minor role for the question of whether the rate of capacity utilisation is driven by a profit-led or a wage-led regime, if a mild elasticity condition on this latter relationship holds true.

2. The simple analytics of the profit rate

In elaborating our argument we consider a Leontief-type macroeconomy with no technical change. By definition the profit rate, \(r\), is then given by the first of the following expressions, and limited in this case by the second one:

\[
r = (1 - v) y(v) - \delta \leq (1 - v) y^p - \delta
\]

Where \(v\) is the wage share, and \(y = Y/K < y^p\) is the rate of capacity utilisation of the capital stock (which is smaller than the potential output-capital ratio, \(y^p\)). The term \(\delta\) is a depreciation rate of a given magnitude. In contrast to the Goodwin model, where the economy is profit-led, i.e. \(y'(v < 0)\), we assume that the economy is wage-led, i.e. \(y'(v) > 0\) holds for the dependence of economic activity on the wage share in the following.

For the derivative of the profit rate with respect to the wage share we obtain:
$r'(v) = -y(v) + (1 - v)y'(v) = -y(v) + (1 - v) \frac{dy}{dv}$  (1)

From this result the following must hold for a positive dependence of the rate of profit on the wage share:

$$\frac{y(v)}{v} < \frac{1 - v}{v} \quad \text{or} \quad \frac{dy}{dv} \frac{1}{y(v)/v} > \frac{1}{1 - v}$$  (2)

Note here that all expressions shown are positive under the assumption of a wage-led economy.

A wage share of 50 per cent is not an unrealistic assumption for advanced capitalist economies (a higher wage share serves our argument better). In order to have positive dependence of the profit rate ($r$) on the wage share in a wage-led regime, we must therefore have an elasticity expression larger than one, see equation (2), i.e. a 1 per cent change in the wage share will increase capacity utilisation by more than 1 per cent. This is a strong reaction, which may happen even with a relatively moderate wage share.

In the actual Goodwin distributive cycle (shown for Germany in Figure 1) this did not happen in the period after 1950. Many economists were surprised by the stagflation that occurred in 1973/74 in Germany, despite Goodwin’s work on this cycle and its implicit suggestion that a profit-squeeze may cause inflation and stagnation. German economic policy, where hydraulic Keynesianism predominated in those times, was unprepared to resolve such Marxian issues (set in the context of the contemporary East-West conflict). Karl Schiller, Professor of Economics and Minister of Economics under Chancellors Kiesinger and Brandt, was a pronounced Hicks-type, not Harrod-type, Keynesian economist who implemented strong stimuli for effective demand to counteract the 1967 recession. This created the illusion of macroeconomic fine-tuning with the goal of full-employment in the economy. However, this led to huge money wage increases in the late 1960s and early 1970s.

---

**Figure 1: The long-phased distributive cycle for Germany after WWII**

(see Groth and Madsen 2013, updated here)
The long stagnation that subsequent German Chancellors – Helmut Schmidt and Helmut Kohl – had to cope with was therefore, at least partially, a result of previous economic policy measures. The subsequent emergence of a second labour market for de-unionised low skilled labour allowed capitalism to become unleashed again, a development further strengthened by the fall of the Iron Curtain and global financialisation. We conclude that a wage-led economy will also be faced, sooner or later, with a decreasing rate of profit if the wage share is increasing. This is a situation which may be tolerated by capital if the rate of profit is exceptionally high, but this will only hold temporarily should the wage share continue to rise. The phase plot of yearly data for Germany after 1950 clearly exemplifies such a situation and its consequences.

We now assert, against common Post-Keynesian practices, that one needs to consider the profit rate and not the profit share as the central variable driving capital accumulation (in relation to the real loan rate). And we judge (as a working hypothesis), from brief calculation and the data shown in the phase plot of Figure 1, that Germany’s rate of profit started to fall in the late 1960s. However, the rate at the beginning was so high that accumulation did not slacken immediately. In the words of Marx (1954 p.431):

Either the price of labour keeps on rising, because its rise does not interfere with the progress of accumulation ...Or, on the other hand, accumulation slackens in consequence of the rise in the price of labour, because the stimulus of gain is blunted. The rate of accumulation lessens; but with its lessening, the primary cause of that lessening vanishes, i.e., the disproportion between capital and exploitable labour power ... The price of labour falls again to a level corresponding with the needs of the self-expansion of capital, whether the level be below, the same as, or above the one which was normal before the rise of wages took place.

We argue that the ‘Either’ in the above quote was true for the period 1967–1972, at most, and that the consequences of this period from the perspective of the private sector, as well as from the misconduct of German Economic policy, were devastating. This is not the central focus of the present paper, but must be left for future empirical and historical research: see also Solow (1990) in this regard.

3. THE DISTRIBUTIVE CYCLE IN A WAGE-LED ECONOMY
We show in this section that the Goodwin model remains qualitatively unchanged as long as its basic Marxian assumptions are intact: (i) that the employment rate drives the value of labour power in a positive fashion, (i.e. in our model the growth rate of the wage share); and, (ii) the rate of profit is the central variable which drives capital accumulation and the growth rate of the employment rate. In this way we formulate a Marx-Keynes (MK) sub-model of the full Goodwin MKS system, where the interest rate may be viewed as a variable in the background of the model, and where labour productivity and the potential output-capital ratio are once again held constant.
On this basis our Goodwin model is derived as follows:

\[ \dot{\nu} = h(e), \ h' > 0, \text{ with a unique } h(e_o) = 0 \]  

\[ \dot{e} = g(r(v)) = i(v), \ g' > 0, \ r' < 0, \text{ with a unique } i(v_o) = 0 \]  

(3)  

(4)  

We restrict our empirical analysis regarding the effective demand function \( y(v) \) to the wage-led case, as discussed in the previous section.  

Turning to the implied dynamics of the system, depicted in equations (3) and (4), it is straightforward to establish the associated phase diagram, as shown in Figure 2. This already indicates the conclusion that can be established for this model (not in graphical terms, but by means of the following calculations), namely that all orbits of the dynamic system considered will be closed curves, i.e. the cycles in employment and wage (share) are neither damped nor explosive (although, empirically, they will generally be much less regular in their shape than that shown in Figure 2).

Figure 2: The Goodwin distributive cycle and its four phases

To prove this assertion we construct a Liapunov function for a system including equations (3) and (4), and make use of the theorem proposed in Hirsch and Smale (1974 p.193) on the stability implications of such a construction. We denote by \( H \) and \( I \) the primitives (integrals) of the functions \( h(e)/e \) and \( -i(v)/v \) for positive \( e \) and \( v \), which fulfil the side conditions \( H(e_o) = I(v_o) = 0 \). Moreover, the sum of these two functions, denoted \( L \), is given as follows:

\[ L : \mathbb{R}^2 \mapsto \mathbb{R}, \ L(v,e) = I(v) + H(e) \]

Where \( L(v_o, e_o) = 0 \). Furthermore, \( L(v, e) > 0 \) if \( (v, e) \neq (v_o, e_o) \).

With regard to the above dynamic system and the function \( L \) we obtain:

\[ \dot{L} = L_e \dot{v} + L_v \dot{e} = (-i(r(v))) \dot{v} + h(e) \dot{e} = (-i(r(v)) + j(y(v)))h(e) + i(v)h(e) = 0 \]

The function \( L \) is therefore constant along the trajectories of the considered dynamics. This implies that the equilibrium \( (v_o, e_o) \) is Liapunov-stable. It should be noted that if we assumed a profit-led economy (and additionally a positive
dependence of investment and the employment rate dynamics on the rate of capacity utilisation) we would get \( \dot{L} < 0 \) (up to the steady state) and:
\[
\dot{L} = L_e \dot{v} + L_v \dot{e} = -i(r(v)) \dot{v} + h(e) \dot{e} = -i(r(v)) h(e) + (i(r(v)) + j(y(v))) h(e) = j(y(v)) h(e) < 0
\]

This is because the first term is negative and the second positive for \( v > v_o \), and vice versa, if the function \( j \) is strictly upward sloping and zero at \( v_o \). A similar calculation also gives this result if \( -k(v) \) is added, as in Blanchard-Katz (1999), to the wage share dynamics \( \dot{v} \).

Furthermore, in this simple scenario, it is not difficult to see that the function \( L \) must be of a form as depicted in Figure 3. This implies that the orbits of the considered dynamics – which are the projections of the level curves of the function \( L \) into the phase plane – must all be closed curves, as in the original Goodwin model shown in Figure 2.  

| Figure 3: The Liapunov function and the implied centre dynamics |

It is easy to show that the distributive cycles become convergent if one augments the first law of motion as follows (see Blanchard and Katz 1999; Barbosa-Filho and Taylor 2006):
\[
\dot{v} = h(e) + k(v), \quad k' < 0, \quad k(v_o) = 0 \quad (5)
\]
as directly implied for the employed Liapunov function \( \dot{L} < 0 \).

Similarly, one might extend the second law of motion as follows:
\[
\dot{e} = i(r(v)) + j(y(v)), \quad j' > 0, \quad j(y(v_o)) = 0 \quad (6)
\]
by assuming, in a by-and-large natural way, that capacity utilisation acts positively on investment in addition to the rate of profit.  This, however, implies \( \dot{L} < 0 \), if the Blanchard-Katz term is sufficiently weak, i.e. the steady state becomes a repeller.
In view of this negative result, we now assume in addition that the effective demand function has a shape as shown in Figure 4.

Figure 4: The aggregate demand function $y(v) = \frac{Y(v)}{K(v)}$ from the global perspective

For very low values of income and capacity utilisation we assume that there is a floor to workers’ consumption, financed by debt, private support etc., but that overall investment reacts positively to the improving profit rate, while the remainder of profits are invested in financial markets (which are exogenous, in the background of the model here). So, the economy is assumed to become profit-led in the situation of a strong wage-squeeze. By contrast, one simply has to assume ceilings in and to eventually get upper turning points. And, for values of the wage-share which approach 1, the economy is again dominated by the evolution of the profit rate. The property $\dot{L} < 0$ therefore holds in general, but not necessarily close-to-the-steady-state of the economy, since we have already shown that a purely profit-led economy would be one with dampened oscillations, i.e. the steady state would then be a global attractor. Starting from a situation which is profit-led for all states of income distribution between capital and labour, two downward sloping segments of the $y(v)$-function to the right and to the left are sufficient to make the economy fluctuate in a cyclical and bounded manner around a steady state, without converging on it.

Based on our knowledge of what happened in Germany between 1974 and 1982, we would suggest that the turning point to the right of the phase plot in Figure 5 (which must take place before the wage-share becomes 100 per cent) may be the subject of historically unique private and public reactions to a strongly increasing wage-share and the strong (mark-up) price inflation implied by it. This turning point can be the subject of theoretical, as well as empirical (historical) analysis. Assuming that effective demand, $y$, falls sufficiently fast when the wage share is approaching 1 (which need not mean that it is close to
one) should bring about the rightwards turn of the overshooting distributive cycle illustrated in Figure 5, without the help of state policy.

**Figure 5: The long wave in income distribution after WWII, Goodwin-type explanation**

The lower turning point, as well as the turning point to the left is, however, generated automatically, even if no policy reaction is occurring, since the dynamic is formulated in terms of growth rates, whereby division by zero becomes impossible.

Despite the fact that our model does not feature many Keynesian feedback mechanisms, our model is able to generate persistent fluctuations which mirror one basic post-war stylised fact of advanced capitalist economies: the clockwise dynamics of the wage share and the employment rate (see again Figure 1).

4. **Discussion**

In our judgement it is no exaggeration to state that the Goodwin growth cycle model represents a prototype which is at least as important as the Solow growth model. Indeed, Solow (1990) himself has expressed his admiration for this compact model. Yet, despite its importance, Goodwin’s model has been largely neglected in the orthodox, as well as in the Post-Keynesian literature. Even more important than this is the observation that the Goodwin model can incorporate the Keynesian theory of effective demand – whether profit- or wage-led – without much change in its qualitative results. This is the case if one accepts that the profit rate (and not the profit share) is the central variable driving capital accumulation. Under this premise the standard Goodwinian Phillips-curve mechanism, together with employment rate dynamics, will confirm the implications of the Goodwin model, independently of whether macroeconomic activity is profit- or wage-led. This is the case if the elasticity of the effective demand function with respect to the wage share is not too large, a condition which is easier to fulfil the higher the wage share becomes. We conclude that Goodwin’s distributive cycle (and Marx’s insight that there is no free lunch for workers under capitalism, as an increasing wage share will
eventually lead to a reduction in the employment level) survives the limited wage-led/profit-led debate, from a theoretical as well as an empirical perspective. These considerations should turn the focus of interest in this debate to the view that the multifaceted Goodwinian MKS-system has to become the focal point for future research.\footnote{Of particular importance, as always, are economic policy issues, as we have already discussed above for the case of Germany. A better policy than the Keynesian fine-tuning approach (misused by Karl Schiller) would have been to avoid the overheating of the economy early enough by using monetary policy. This – to a certain degree – accepts Milton Friedman’s policy advice, but does this from a Marx-based view regarding the wage-price spiral, the so-called NAIRU, and the reserve army mechanism. Moreover, institutionally prescribed lower (and upper) bounds to the wage share (including, therein, manager salaries and bonuses) may have reduced the overshooting employment and unemployment rates, as shown in Figure 6 (see Flaschel and Greiner 2009; Flaschel et al 2011 for details).}

Of particular importance, as always, are economic policy issues, as we have already discussed above for the case of Germany. A better policy than the Keynesian fine-tuning approach (misused by Karl Schiller) would have been to avoid the overheating of the economy early enough by using monetary policy. This – to a certain degree – accepts Milton Friedman’s policy advice, but does this from a Marx-based view regarding the wage-price spiral, the so-called NAIRU, and the reserve army mechanism. Moreover, institutionally prescribed lower (and upper) bounds to the wage share (including, therein, manager salaries and bonuses) may have reduced the overshooting employment and unemployment rates, as shown in Figure 6 (see Flaschel and Greiner 2009; Flaschel et al 2011 for details).

Figure 6: The distributive cycle with some labour market regulation
(through cooperation between capital and labour or legislation)

We view the period 1945–1998 (approximately) as the fourth long wave of capitalism, after Marx’s analysis of the Industrial Revolution, Schumpeter’s Bourgeois Kondratieff, and Keynes’s European Wartime wave. The transition into the new century may, however, have been the starting point of a new type of wave, one in which the present analysis may find no application anymore; one which might be thought of as a ‘glassy wave’, for lack of a better term (from 1998 the internet became a commercial issue). Prior to this, however, was a wave of mass consumption, of five stages in the evolution of democracy, but also the time of the rise and fall of the welfare state in Europe, as well as the rise and the fall of the Iron Curtain. In Figure 7 we see (for the UK) the short waves in the distributive cycle before unionisation became particularly effective, the wartime wave with its big depression (which brought Hitler to power), and the wave after
WWII with its long-term distributive cycle similar, but not so well-shaped as the one for Germany, where the rise in the welfare state was more pronounced.

Figure 7: A possibly exceptional long distributional wave after WWII (exemplified by UK data from Groth and Madsen).
UKER: UK employment rate, UKWS: UK wage share

5. Conclusions
The reader may ask what the foregoing analysis has to do with the simple Goodwin model of the distributive cycle, whether wage-led or not. The answer is that there is an urgent need to develop further the grand visions of Marx, including those that have proven to be robust over time, namely the reserve army mechanism, and the conflict over income distribution between capital and labour in its shaping of the social structure of capital accumulation. Moreover, we need to consider further the conflict between capital and labour in the sphere of production, the law of falling labour content – under capital-using labour-saving technical change (see Flaschel et al 2013) – and how this conflict may affect the functioning of democratic institutions and the very sustainability of our modern capitalist societies.

Accepted for publication: 17 February 2018

endnotes
1. Matthieu Charpe, International Labour Organization (ILO), Switzerland. Email: charpe@ilo.org. Peter Flaschel, Bielefeld University, Germany. E-mail: pflaschel@wiwi.uni-bielefeld.de. Christian R. Proaño, University of Bamberg, Germany. Email: christian.proano@uni-bamberg.de.

2. Of course, financial markets (in particular risk-taking credit) also play a central role in Schumpeter’s thinking, while Marx concentrates more on the production of absolute
and relative surplus value – the exploitation of labour power and the law of value underlying it – and on its distribution between industrial and financial capitalists.

3. There is a decoupling of ‘utilisation’ of the employed labour force (over- and under-time work) from the law of motion of the employment rate implicitly involved in the above equation system, as was pointed out to us by Michalis Nikiforos. Indeed, the rate of utilisation of the insiders does not yet have any impact on the considered dynamics which in this respect is therefore incomplete and must be extended, as for example done by Flaschel (2000): see also his later work on this matter. In the present model, utilisation of the workforce of firms varies positively with \( v \), as does the growth rate of the employment rate of the whole workforce, i.e. the former business cycle effect runs counter to the long-phased reserve army mechanism, which is driven primarily by the evolution of the profit-rate. There is thus a need to introduce feedback effects between the utilisation rate of the workforce of firms and the labour market employment rate. Moreover, the wage-led effect in the business cycle may only be a temporary one.

4. \( \dot{L} \), the derivative along the trajectories of the system, and \( L_v, L_e \) the partial derivatives of the Liapunov function \( L \).

5. Note here however that the profit rate function is nonlinear by definition.

6. This extension has been suggested to us by Gangolf Groh.

7. Of course, the Goodwin model has been already extended and modified in many ways (see Flaschel 2015 for a survey regarding Goodwin and the MKS system).

**References**


