Hyperbolic Policymakers and Economic Growth

Diego Nocetti

ABSTRACT

In this note I explore how a non-constant rate of time preference on the part of policymakers affects economic growth. In a simple dynamic general equilibrium model I show that if the incumbent government has a rate of time preference in the form of a quasi-hyperbolic discounting function, tax rates can be substantially higher and economic growth considerably lower than the standard case of exponential discounting.

1. INTRODUCTION

The hypothesis that policymakers set economic policies to maximize the constituency's welfare is a maintained assumption in the vast majority of growth models. This assumption is attractive because it avoids the myriad modeling choices that arise once altruistic interests are removed. However, as Benjamin and Laibson (2003, p.1) note 'Politicians and policymakers are prone to the same biases as private citizens. Even if politicians are rational, little suggests that they have altruistic interests.'

Motivated by the work of Strotz (1956), and based on experimental findings, recent research has studied the implications of higher rates of time preference in the short-run and in particular of hyperbolic discounting on the part of consumers (e.g. Harris and Laibson, 2001). Notwithstanding the general belief that politicians in office put a much higher weight on current relative to future outcomes, the implication of hyperbolic discounting has not received the same attention in the political economy literature. The objective of this note is to start filling this gap by exploring the link among growth, policy instruments, and policymakers' impatience.

I develop a simple dynamic general equilibrium model in which the incumbent government sets fiscal policy to maximize the utility of a represen-
tative household, but, as opposed to them, has a non-constant rate of time preference in the form of a quasi-hyperbolic discounting function. This simple departure from the standard case of exponential discounting has important implications. I show that tax rates can be substantially higher and economic growth considerably lower than the standard case of exponential discounting.

In the next section I present the basic structure of the economy and the equilibrium conditions from the perspective of a representative consumer, taking tax policy as given. In section 3 I analyze Markov tax policy strategies for a policymaker that discounts utility flows with a non-constant rate of time preference and compare the resulting Markov perfect equilibrium with that under exponential discounting. Section 4 concludes.

2. The Private Sector
I consider a closed economy populated by a representative agent and a government sector. The consumer maximizes lifetime utility over an infinite planning horizon and discounts utility flows exponentially. He derives utility from private and government consumption and, for simplicity, I assume that preferences are time separable and the felicity function is logarithmic.

\[ U_t = \sum_{r=0}^{\infty} \beta^r (\log c_t + \delta \log g_t) \]  

(1)

where \( \delta \) is the relative weight on government consumption.

The household-producer has access to a Cobb Douglas technology

\[ y_t = A k_t^\alpha \]  

(2)

where \( y \) is output per worker and \( k \) is capital per worker with \( k_0 \) given. Since I abstract from labour, \( k \) should be interpreted broadly to include human as well as physical capital (Rebelo, 1991).

By equalizing the interest rate to the marginal product of capital, and for simplicity assuming full capital depreciation, the household budget constraint can be written as

\[ c_t = (1 - \phi) \alpha A k_t^\alpha - k_{t+1} \]  

(3)

where \( \phi \) is the income tax rate.

I assume that each period the government runs a balanced budget and its revenues come exclusively from income taxes. Thus, the government budget constraint must satisfy

\[ g_t = \phi \alpha A k_t^\alpha \]  

(4)

Taking tax policy as given, the state variables from the households’ perspective are the beginning of the period capital stock and the current tax rate. Thus, the value function \( V(k_t, \phi_t) \) must satisfy the Bellman equation

\[ -42 - \]
subject to (3).

Given the simple structure of the problem it is simple to show that, for Markov tax strategies, in a competitive equilibrium, optimal consumption and the dynamics of the capital stock are characterized by the following equations (see Appendix)  

\[ c_t = (1 - \beta)(1 - \phi) \alpha Ak_t^{\alpha} \]  
\[ k_{t+1} = \alpha \beta (1 - \phi) Ak_t^{\alpha} \]  

Equations (6) and (7) imply that both consumption and capital accumulation decrease with the current tax rate. Intuitively, an increase in the tax rate reduces disposable income and the rate of return on capital.

3. OPTIMAL POLICY

I assume that policymakers stay in power for one period and set policy to maximize the representative agent’s utility. However, they care relatively less about future outcomes than do consumers. This does not mean, however, that policymakers fail to account for the long term consequences of their actions. In particular, I assume that policy is set to maximize households’ utility using a quasi-hyperbolic discounting function.

\[ U_t = \log c_t + \delta \log g_t + \lambda \sum_{t=0}^{\infty} \beta^t \left( \log c_{t+1} + \delta \log g_{t+1} \right) \]  

where \( 0 < \lambda < 1 \) captures the property that the short run discount rate is greater than the long run discount rate.

Note that the set-up is closer to Phelps and Pollak (1968), who use hyperbolic discounting to approximate intergenerational utility flows, than that of Harris and Laibson (2001) who use the function in an intrapersonal setting.

I analyze the set of Markov strategies of a game among policy-makers at different time periods and a Stackelberg game between the incumbent government and the representative agent. Specifically, each time period the policymaker in power selects Markov tax policy strategies to maximize (8) given (6), (7), and the choices of future policy-makers. The solution for \( \phi_t \), together with (6) and (7), results in a Markov-perfect general equilibrium.

Consider a policymaker that has to set the tax rate at time \( t \). For him, the only state variable is the stock of capital. His current-value function can be written as follows

\[ W(k_t) = \max_{k_t} \left[ \log c_t + \delta \log g_t + \lambda \beta V(k_{t+1}) \right] \]  

- 43 -
where \( V(kt) \) is his continuation value function that discounts utility flows exponentially and solves the recursive equation
\[
V(kt+1) = \log c_{t+1} + \delta \log g_{t+1} + \beta V(k_{t+2})
\] (10)
and the maximization is subject to (4), (6), and (7).

Note that \( W \) and \( V \) are linked by the following equation
\[
\lambda V(kt) = W(k_{t+1}) - (1 - \lambda)(\log c_{t+1} + \delta \log g_{t+1})
\] (11)
from which it follows that the Bellman equation satisfies
\[
W(k_t) = \max_\delta \left[ \log c_t + \delta \log g_t + \beta W(k_{t+1}) - \beta (1 - \lambda)(\log c_{t+1} + \delta \log g_{t+1}) \right]
\] (12)
The difference between (12) and the exponential case is the term
\[-\beta (1 - \lambda)(\log c_{t+1} + \delta \log g_{t+1})\]
This term represents, as Harris and Laibson (2003) note, the expected value of a change in \( W \) that occurs in period \( t+1 \) due to the arrival of a new decision-maker.

The derivation of the optimal tax rate is straightforward (see Appendix)
\[
\phi = \phi = \delta \overline{\delta + \Omega}
\] (13)
where
\[
\Omega = 1 + \frac{(1 + \delta)\alpha\beta}{1 - \alpha\beta} \left[ \frac{1 - (1 - \lambda)(1 + \beta(2 - \alpha))}{1 - \alpha\beta} \right]
\]
Consider the properties of the optimal policy. The tax rate, or equivalently the expenditures-to-output ratio, is constant over time. Even for impatient policymakers, the desire to minimize distortions leads them to smooth the path of taxes over time.

The term \( \Omega \) is the effective discount rate. In the absence of hyperbolic discounting the tax rate reduces to
\[
\phi = \phi = \frac{\delta (1 - \alpha\beta)}{1 + \delta}
\] (14)
which is obtained by setting \( \lambda \) to one in (13). Comparing equations (13) and (14) it is evident that hyperbolic discounting reduces the effective discount rate and, therefore, the optimal tax rate is higher compared to the exponential case. In turn, by (6) and (7), consumption and growth are lower under hyperbolic discounting relative to exponential discounting.
As in Barro (1999) and Palacios Huerta (2003), since $\Omega$ is constant, the Markov-perfect equilibrium is observationally equivalent to the exponential case for a suitable choice of the discount factor. Thus, since the discount factor is not observed directly, there is a problem in trying to infer from the data whether different tax rates are due to consumers’ preferences or due to the policymaker’s incentive to produce short run results.

How much higher can the tax rate be under hyperbolic discounting? Figure 1 shows that the tax rate can be substantially higher. For example, given a realistic parameter configuration, the figure shows that for $\lambda = 0.5$ the tax rate increases from 0.25 [the standard case given in (14)] to 0.38. These numbers, however, should be treated with care. As Harris and Laibson (2003) note, discrete time models with hyperbolic discounting generate a number of pathologies in consumption decisions, such as consumption functions that are not globally monotonic in wealth. Our model is not immune from those problems. Inspection of (13) shows that the tax rate can drop discontinuously and may even be negative. Although, this happens only in a limited region of the parameter space, one should at least be cautious in drawing implications from the absolute values obtained.

**Figure 1: Tax rate for different values of discounting**

![Hyperbolic Discounting vs. Exponential Discounting](image)

4. **Conclusions and Extensions**

In this note I have shown that introducing a non-constant rate of time preference into the policymakers’ problem can lead to a highly inefficient allocation of resources and substantially affect economic growth. From a normative perspective, the results imply that a more efficient outcome would result if policymakers had a lower weight in government consumption (to counteract their impatience) than the representative consumers (i.e. a lower $\delta$). This is similar
to Rogoff's (1985) monetary model where equilibrium inflation can be reduced by appointing a conservative and independent central banker. Alternatively, establishing caps on the expenditure-to-output ratio could potentially mitigate policymakers' impatience. This, of course, leads to the well known policy dilemma of choosing between rules and discretion.

From a positive point of view, the results reported above might contribute to explaining why different countries pursue different tax policies, and how these differences affect growth, under similar economic conditions. In this sense, policymakers' impatience in the form of a hyperbolic discounting function complements the findings of the political business cycle literature, whereby short-sighted policies lead to highly inefficient resource allocations.

The model is simple and there are several areas to pursue extensions. First, I assume that policymakers stay in office for one period, disregarding electoral uncertainty. One might argue that electoral processes represent a commitment device and therefore might lead to better outcomes. Introducing electoral uncertainty can readily be addressed. For example, suppose that the arrival of a new policymaker in the next period occurs with a probability \( p \) and that this probability is independent of the time the incumbent government has been in power. Further, assume that policymakers have a higher discount rate only if they are not in power. Then, the incumbent government maximizes

\[
U_t = \mu(c_t, g_t) + p\lambda \sum_{i=1}^{\infty} \beta^i \mu(c_{i+1}, g_{i+1}) + (1-p) \sum_{i=1}^{\infty} \beta^i \mu(c_{i+1}, g_{i+1})
\]

In this case, it is the interaction between electoral uncertainty and hyperbolic discounting that matters. If either the policymaker discounts exponentially, or she is reelected with certainty every period in the future (\( p=0 \)), the equilibrium outcome will result in the consumer's optimum. Therefore, electoral uncertainty decreases growth compared to the case of no uncertainty, but it also mitigates the effect of hyperbolic discounting since the policymaker will set taxes considering the possibility that she will remain in power in the future.

A second drawback of the model is that I treat the government as a single entity. In reality, policymaking is a complex process shaped by the interaction of individuals and parties that might differ in their current and future objectives. For example, the incentive of the incumbent government to have a high tax rate might be mitigated by the existence of an opposing party, one that cares relatively more about future economic outcomes when they might be in power. This strategic interaction among policymakers as a group and the pervasive effects of their impatience certainly deserves more attention.

Accepted for publication: 20 December 2005
APPENDIX

The consumer’s problem has a simple structure and can be solved using dynamic programming. In particular, given the log-linear felicity function postulate a solution to the Bellman equation of the form

\[ V(k_\tau, \phi_\tau) = a_0 + a_1 \log k_\tau + a_2 \log \phi \]  
(A.1)

where the coefficients \( a_0, a_1, \) and \( a_2 \) are to be determined.

The optimality condition sets the marginal utility of consumption equal to the discounted marginal utility of wealth. Using the conjecture this is given by the equality

\[ \frac{1}{c_i} = \beta \frac{a_i}{k_{i+1}} \]  
(A.2)

The envelope theorem implies

\[ V_i(k_\tau, \phi_\tau) = \frac{(1-\phi_\tau)r_i}{c_i} \]

which, using the equality of the interest rate to the marginal product of capital and the conjecture, implies

\[ \frac{a_i}{k_i} = \frac{(1-\phi_\tau)\alpha \Lambda k_i^{\alpha-1}}{c_i} \]  
(A.3)

Using (A.2) and (A.3) gives (7) in the text. In turn, using (7) and the budget constraint gives (6).

Following the logic above, to solve the policy-maker’s problem guess that the solution to the Bellman equation is log-linear in the level of capital

\[ V^p(k_\tau) = a_0^p + a_1^p \log k_\tau \]  
(A.4)

Differentiating the right hand side of (12) with respect to \( \phi_\tau \) using (A.4) to plug the implied values of \( \phi_\tau \) back into the Bellman equation, and equating coefficients gives (13).

ENDNOTES

1. 400 BB, Fogelman College of Business & Economics, The University of Memphis, Memphis, TN 38152, USA. Phone: (901) 759-0427. E-mail: dnocetti@memphis.edu I am grateful to an anonymous referee for very helpful comments.

2. The parameter values are as follows: \( \alpha = 0.25, \beta = 0.99, \delta = 0.5 \).


